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Optimization of the polynomial fifth-order interpolation 2P kernel in the time-domain

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Abstract. The paper describes an algorithm to optimize fifth-order polynomial interpolation two-parameter (2P) kernel r in the time-domain. The optimization criterion is the minimization of interpolation error e. The result of the optimization is kernel $r^t_{opt,2P}$, with the optimal value of kernel parameters ($\alpha_{opt} = 98/2707$, $\beta_{opt} = -113/8934$). An experiment is made to determinate the interpolation precision when using the $r^t_{opt,2P}$ kernel in relation to fifth-order kernels 1P ($r^f_{opt,1P}$) and 2P ($r^f_{opt,2P}$) which are optimized in the spectral-domain. The mean-square error is used as a measure of the interpolation precision. A comparative analysis of the results shows that the interpolation precision, when interpolating with the 2P kernel is by 1.0542 and 1.0133 times higher than the interpolation precision when using the 1P and 2P kernels, respectively. The interpolation execution time with the 2P kernel being $t_{exe} = 9.2341 \cdot 10^{-8}$ s, the optimized 2P fifth-order kernel can be recommended for the implementation in a real-time system.

Keywords: Convolution, Interpolation, Polynomial kernel, Taylor series

Optimizacija polinomskega jedra interpolacije petega reda 2P v časovni domeni

Prispevek opisuje algoritem za optimizacijo jedra r polinomske interpolacije petega reda z dvema parametroma (2P) v časovni domeni. Optimizacijski kriterij je bila minimizacija interpolacijske napake e. Rezultat optimizacije je jedro $r^t_{opt,2P}$ z optimalno vrednostjo parametrov jedra ($\alpha_{opt} = 98/2707$, $\beta_{opt} = -113/8934$). Izvedli smo eksperiment, katerega namen je bil določiti natančnost interpolacije pri uporabi jedra $r^t_{opt,2P}$ glede na jedro petega reda 1P ($r^f_{opt,1P}$) in 2P ($r^f_{opt,2P}$), ki sta optimizirana v spektralni domeni. Kot merilo natančnosti interpolacije je bila uporabljena srednja kvadratna napaka (MSE). Primerjalna analiza rezultatov je pokazala, da je natančnost interpolacije pri interpolaciji z jedrom 2P, ki je optimizirano v spektralni domeni, večja od natančnosti interpolacije pri uporabi jeder 1P in 2P, in sicer 1,0542 oziroma 1,0133-krat. Glede na to, da je čas izvajanja interpolacije z jedrom 2P $t_{exe} = 9.2341 \cdot 10^{-8}$ s, je optimizirano jedro petega reda 2P priporočljivo za implementacijo v sistemu realnega časa.

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1 INTRODUCTION

The interpolation is a very important task in the digital signal processing [1]. For example, in the digital speech processing, the resampling and estimation of the fundamental frequency and the speaker's emotional and health state, etc. are performed [2]. In the field of the musical signal processing, in addition to the filtering, extraction and transcription of the solo and bass lines, recognition of chords and their transcription [3], evaluation of the parameters of the played tone (intonation, vibrato rate, vibrato extend), etc. are carried out [4]. In the digital image processing, spatial transformations (resampling, image dimension change, rotation, geometric deformation, etc.) are often performed [5], [6], [7]. To realize the spatial transformations, it is necessary to determine the pixels whose location is outside the grid [8], [9], [10]. These are just some of the examples where it is necessary to apply the numerical method, known as interpolation [11], [12].

For the interpolation purposes, it is possible to use the well-known numerical interpolation formulea (Lagrangian, Newtonian, Gaussian, Stirling, Bessel, Chebyshev,...) [13]. However, it is necessary to use in that case a large number of samples, which leads to a large order of the interpolation function. As a result, the numerical complexity increases, which results in a long interpolation execution time. Algorithms that have a long execution time are not suitable for the implementation in real-time systems.

In order to reduce the interpolation execution time, the convolution interpolation is intensively applied. In the convolution interpolation, convolution is performed between the discrete signal and the interpolation kernel. The ideal interpolation kernel is function $\sin(x)/x$, which is symbolically denoted as *sinc*. The *sinc* kernel is defined on interval $(-\infty, +\infty)$, and its spectral characteristic is rectangular, that is, a *box* function. The properties of the *box* spectral characteristic are: *a*) in the pass-band, it is flat and equal to one, *b*) in the stop-band, it is flat and equal to zero, and *c*) with an ideal slope in the transition area [14]. However, due to its infinite length $(L \to \infty)$, the *sinc* kernel cannot be realized [15]. A simple solution is a truncate *sinc* kernel, and thus, the length of kernel L becomes finite. Truncating of the kernel is performed using a rectangular window of length L. This process is known as a windowization. In this way, an interpolation kernel of length L is formed. Therefore, the spectral characteristic of the truncated *sincw* kernel is distinguished from the spectral *box* characteristic (the spectral characteristic has a ripple in the pass-band and the stop-band, as well as a finite slope in the transition range). The consequence of the interpolation with a *sincw* kernel is a decrease in the interpolation precision. In order to increase the interpolation precision, an intensive work is being done on the creation of interpolation kernel r, length L, which should have the spectral characteristic that closely approximates the spectral *box* function. Interpolation kernel r should be the simplest, that is, it should be created from a relatively simple mathematical function, in order to reduce the interpolation execution time. Today, the interpolation convolution kernels that are created from the low degree polynomials $(n \leq 7)$ are very current [16].

A large number of the polynomial kernels have been proposed in the scientific literature [17]. Numerically the simplest one is the polynomial zeroth-order kernel. The interpolation is performed by rounding to the nearest-neighbour sample [15], [18]. Besides, the high execution speed, the interpolation with this kernel leads to the appearance of large interpolation error e. A linear, polynomial first-order interpolation kernel is described in [19], [20]. A cubic polynomial third-order interpolation kernel is described in [16]. The convolution interpolation which uses the third-order kernel is more precise than the former two kernels. The parameterization of the polynomial third-order kernel is proposed by Robert Keys in [21]. By inserting parameter α into the coefficients of the kernel, the parameterization is performed. A very significant fact is that by changing a kernel parameter α, the kernel can be adapted to a specific signal, and in this way, the interpolation precision can be increased. In the scientific literature, this kernel has been in honour of its author named after him. In order to further increase the precision of the interpolation, kernels of the third-order with two parameters (2P Keys, parameters α and β) are proposed [22]. In [23], the interpolation precision in estimating the fundamental frequency of the speech signal, using the 1P and 2P interpolation kernels, is analyzed. The analysis of the results shows that interpolation error MSE, when applying the 2P kernel, is by 2,65 times smaller. A further increase in the interpolation precision is achieved by constructing a three-parameter (α, β, γ) 3P Keys kernel [19]. The analysis of the results shows that interpolation error MSE, when estimating the fundamental frequency of the speech signal using 3P kernel ($\alpha_{opt} = 0.9$, $\beta_{opt} = -0.8$, $\gamma_{opt} = -3.1$), is by 7,74 times higher than when using the 1P kernel ($\alpha_{opt} = -1.02$) and 7,28 times higher than in using the 2P kernel ($\alpha_{opt} = -0.55$, $\beta_{opt} = 1.2$).

The polynomial fifth-order one-parameter interpolation kernel is described in [24]. Its length is L = 6. The optimization of kernel parameter α in the spectral-domain is performed. The optimization criterion is the reduction of the ripple of the spectral characteristic in the pass-band and the stop-band. Through the optimization process, the optimal kernel parameter, $\alpha_{opt} = 3/64$, is determined. In [25], the parameterization of a two-parameter fifth-order interpolation kernel of length $L = 8$ is described. The analysis of the interpolation error, when the standard test images (Lena, Barbara, Camerman, ...) are interpolated, shows that the error is by 1,07 times smaller when using the 2P kernel ($\alpha_{opt} = -0.025$, $\beta_{opt} = -0.0562$) than when using of the 1P kernel ($\alpha_{opt} = 0.0313$). The spectral characteristics of the kernel are described in [26]. The optimization of the kernel is performed in the spectral-domain [27]. The optimization criterion is the minimization of the ripple of the amplitude spectral characteristic. In this way, the optimal values of the kernel parameters of the fifth-order, $\alpha_{opt} = 171/1408$ and $\beta_{opt} = 525/7744$, are determined. In [21], the optimization of the polynomial third-order 1P kernel in both the spectral and time-domains is presented. A detailed analysis shows that the optimal values of the kernel parameters in both domains are equal, that is, α^f_{opt} = α^t_{opt} = -0.5. In the scientific literature, the optimization of the polynomial fifth-order 2P kernel has not been presented so far.

The paper presents results of an optimization of the fifth-order 2P kernel in the time-domain. As an optimization criterion, the minimization of interpolation error e is applied. The first part presents the optimization algorithm. First, interpolation function g is determined. Assuming that function f, which is to be interpolated, has at least five continuous derivatives in the interval where the interpolation is performed, the development of function f in the Taylor series is performed. The Taylor series is expanded to the fifth term. Then, interpolation error $e = f - g$ is made. Finally, the minimization of the interpolation error is realized, so that f and interpolated function g agree up to the fifth term in the Taylor series expansion. The minimization of the interpolation error is achieved by minimizing the fourteen coefficients in the Taylor series expansion. In this way, a system of fourteen equations with two variables is set up. As this case it is not possible to find a unique solution, the least-square method (LSM) is applied. As a result, optimal kernel parameters α_{opt} and β_{opt} are calculated. To verify the correctness of the choice of the optimal kernel parameters, an experiment is carried out. First, for the purposes of the experiment, the test signal base is created. The database is composed of the test signals classified into four groups: a) Math test signals (*f*) formed by mathematical functions with a complex time form; b) noise Math test signals (f_n) created by superimposing the white Gaussian noise (WGN) on the Math test signals, and, in this way, the signal-to-noise ratio (SNR) in the range of SNR = 10 - 70 dB is achieved; c) music test signals (s_G), which are obtained by recording G tones (G1 - G7) that are interpreted on a Steinway B piano. The music test signals are a part of the RWC music database created at the University of Iowa [28]; and d) speech test signals (*s*) are taken from the EMODB database of the German Emotional Speech created by the Institute of Communication Science, Technical University, Berlin, Germany [29]. After that, the test signals are interpolated using the convolution interpolation with the fifth-order kernels: a) the 1P kernel [24] and b) the 2P kernel [27] whose optimizations are performed in the spectral-domain, where the optimization criterion is the minimization of the ripple of the amplitude spectral characteristic. Then, interpolation errors e and the mean square errors MSEs are calculated. Finally, a comparative analysis of the interpolation precision of the kernel that was optimized in the paper, using optimization in the time-domain, with kernels whose optimizations are performed in the spectral-domain, is performed. As a measure of the interpolation precision, MSE is used. The results of the experiment are presented in graphs and tables.

The paper is further organized as follows. Section 2 describes the fifth-order 2P interpolation kernel, Section 3 the kernel optimization in the time-domain, Section 4 an experiment with a comparative analysis of its results and Section 5 Conclusion.

2 Fifth-order interpolation kernels

2.1 2P Kernel

Paper [25] describes a polynomial fifth-order two-parameter kernel. The 2P kernel is defined on interval (-4, 4) and approximates the ideal *sinc* interpolation kernel. Outside the interval (-4, 4), the interpolation kernel is zero. The 2P kernel is composed of the piecewise fifth-order polynomials which are defined on the subintervals (-4, -3), (-3, -2), (-2, -1), (-1,0), (0,1), (1, 2), (2, 3) and (3, 4). Therefore, the length of the kernel is $L = 8$. Kernel r is defined by:

$$r(s) = \begin{cases} a_{50}|s|^5 + ... + a_{10}|s| + a_{00}, & |s| \le 1 \\ a_{51}|s|^5 + ... + a_{11}|s| + a_{01}, & 1 < |s| \le 2 \\ a_{52}|s|^5 + ... + a_{12}|s| + a_{02}, & 2 < |s| \le 3 \\ a_{53}|s|^5 + ... + a_{13}|s| + a_{03}, & 3 < |s| \le 4 \\ 0, & \text{otherwise} \end{cases}, \quad (1)$$

where $a_{50} = 10\alpha - 10\beta - 21/16$, $a_{40} = -18\alpha + 18\beta + 45/16$, $a_{30} = 0$, $a_{20} = 8\alpha - 8\beta - 5/2$, $a_{10} = 0$, $a_{00} = 1$, $a_{51} = 11\alpha - 11\beta - 5/16$, $a_{41} = -88\alpha + 88\beta + 45/16$, $a_{31} = 270\alpha - 270\beta - 10$, $a_{21} = -392\alpha + 392\beta + 35/2$, $a_{11} = 265\alpha - 265\beta - 15$, $a_{01} = -66\alpha + 66\beta + 5$, $a_{52} = \alpha$, $a_{42} = -14\alpha + 3\beta$, $a_{32} = 78\alpha - 30\beta$, $a_{22} = -216\alpha + 112\beta$, $a_{12} = 297\alpha - 185\beta$, $a_{02} = -162\alpha + 114\beta$, $a_{53} = \beta$, $a_{43} = -19\beta$, $a_{33} = 144\beta$, $a_{23} = -544\beta$, $a_{13} = 1024\beta$, $a_{03} = 0$, and α and β are the kernel parameters. Kernel parameters α and β directly affect the time-spectral characteristics of the 2P kernel (1). Changing the value of the kernel parameters affects the interpolation precision. By minimizing interpolation error e, the optimal value of the kernel parameter is determined, and in this way, interpolation kernel r is optimised. The interpolation kernel can be optimised in: *a*) the spectral and *b*) the time-domain. Optimization in the time-domain implies the minimization of the difference between the amplitude spectral characteristics of ideal kernel *sinc*, whose characteristic is the *box* function, H_{sinc}, and the analyzed interpolation 2P kernel r, whose spectral characteristic is H. The paper [27] describes the optimization of the 2P kernel in the spectral-domain ($\alpha_{opt} = 171/1408$ and $\beta_{opt} = 525/7744$). The optimization criterion is the minimization of the ripple of spectral characteristic H. The interpolation kernel optimized in the spectral-domain is denoted by r^f_{opt}, and its spectral characteristic by H^f_{opt}.

The rest of the paper presents the optimization of the polynomial 2P kernel performed in the time-domain. The optimization criterion is the minimization of interpolation error e.

3 Optimization algorithm in the time-domain

The optimization of the fifth-order polynomial 2P kernel is a process in which the optimal values of the kernel parameters alpha and beta are determined. The optimization is performed in the time-domain, by minimizing the interpolation error. The optimization algorithm is implemented in the following steps:

Input: $r(\alpha, \beta)$ - 2P interpolation kernel, f - signal for interpolation.
Output: α_{opt}, β_{opt} - the optimal kernel parameters.
Step 1: Determination of interpolation function g in accordance with the definition of the convolution interpolation,
Step 2: Determination of the kernel r value for each segment in which the kernel is defined,
Step 3: When function f has at least five continuous derivatives in the analyzed interval, the expansion of function f into the Taylor series is performed,
Step 4: The expansion of interpolation function g into the Taylor series up to the fifth term is carried out,
Step 5: Determination of interpolation error e,
Step 6: Minimizing of the interpolation error e using LSM and determination of the optimal values of kernel parameters $alpha_{opt}$ and $beta_{opt}$.

A detailed description of each step is given below:

Step 1: Interpolation function $g(x)$ is a special type of the approximation function. Its basic property is that it is equal to the sampled data, that is, the values of function $f(x)$ in the interpolation nodes. Then $g(x_k) = f(x_k)$, where $0 \leqslant k \leqslant N-1$, and N is the total number of the interpolation nodes in the segment where the function is interpolated. Let us assume that x is a point in which the interpolation of function $f(x)$ should be performed. Let x be between the two consecutive interpolation nodes denoted as x_j and x_{j+1}. Let $s = (x - x_j)/h$, where h is the sampling increment. Then $(x - x_k)/h = (x - x_j + x_j - x_k)/h = s + j + k$. The interpolation, i.e. the reconstructed function $g(x)$ is determined by the convolution interpolation [21], [30] of interpolation function $f(x)$ with the interpolation kernel r:

$$g(x) = \sum_k c_k r\left(\frac{x - x_k}{h}\right) = \sum_k c_k r(s + j - k), \tag{2}$$

where c_k is the value of the function $f(x)$ in the interpolation k-th node (k-th sample), and h is the sampling increment. By developing the sum from (2), the reconstruction function can be written as:

$$\begin{aligned} g(x) &= c_{j-3} r(s+3) + c_{j-2} r(s+2) \\ &+ c_{j-1} r(s+1) + c_j r(s) + c_{j+1} r(s-1) \\ &+ c_{j+2} r(s-2) + c_{j+3} r(s-3) \\ &+ c_{j+4} r(s-4). \end{aligned} \tag{3}$$

Step 2: The value of kernel r for segment $-4 \leqslant s < -3$, is:

$$r(s+3) = \beta s^5 - 4\beta s^4 + 6\beta s^3 - 4\beta s^2 + \beta s. \tag{4}$$

By continuing the procedure, the kernel values in other segments are determined: a) $-3 \leqslant s < -2$: $\Rightarrow r(s+2) = \alpha s^5 + (3\beta - 4\alpha) s^4 + (6\alpha - 6\beta) s^3 + (4\beta - 4\alpha) s^2 + (\alpha - \beta) s$; b) $-2 \leqslant s < -1$: $\Rightarrow r(s+1) = (11\alpha - 11\beta - 5/16) s^5 + (33\beta - 33\alpha + 5/4) s^4 + (28\alpha - 28\beta - 15/8) s^3 + 5/4 \cdot s^2 + (6\beta - 6\alpha - 5/16) s$; c) $-1 \leqslant s < 0$: $\Rightarrow r(s) = (10\alpha - 10\beta - 21/16) s^5 + (18\beta - 18\alpha + 45/16) s^4 + (8\alpha - 8\beta - 5/2) s^2 + 1$; d) $0 \leqslant s < 1$: $\Rightarrow r(s-1) = (10\beta - 10\alpha + 21/16) s^5 + (32\alpha - 32\beta - 15/4) s^4 + (28\beta - 28\alpha + 15/8) s^3 + 5/4 \cdot s^2 + (6\alpha - 6\beta + 5/16) s$; e) $1 \leqslant s < 2$: $\Rightarrow r(s-2) = \left(11\beta - 11\alpha + \frac{5}{16}\right) s^5 + (22\alpha - 22\beta - 5/16) s^4 + (6\beta - 6\alpha) s^3 + (4\beta - 4\alpha) s^2 + (\beta - \alpha) s$; f) $2 \leqslant s < 3$: $\Rightarrow r(s-3) = -\alpha s^5 + (\alpha + 3\beta) s^4 - 6\beta s^3 + 4\beta s^2 - \beta s$; and g) $3 \leqslant s < 4$: $\Rightarrow r(s-4) = -\beta s^5 + \beta s^4$.

By substituting it in (2), the interpolation function is written in the form:

$$\begin{aligned} g(x) = \big(&\beta c_{j-3} + \alpha c_{j-2} + D_{5,-1} c_{j-1} + D_{5,0} c_j \\ &+ D_{5,1} c_{j+1} + D_{5,-2} c_{j+2} - \alpha c_{j+3} - \beta c_{j+4}\big) s^5 \\ &+ \big(-4\beta c_{j-3} + D_{4,-2} c_{j-2} + D_{4,-1} c_{j-1} \\ &+ D_{4,0} c_j + D_{4,1} c_{j+1} + D_{4,2} c_{j+2} \\ &+ D_{4,3} c_{j+3} + \beta c_{j+4}\big) s^4 \\ &+ \big(6\beta c_{j-3} + D_{3,-2} c_{j-2} + D_{3,-1} c_{j-1} \\ &+ D_{3,1} c_{j+1} + D_{3,2} c_{j+2} - 6\beta c_{j+3}\big) s^3 \\ &+ \big(-4\beta c_{j-3} + 4 D_{2,-2} c_{j-2} + \frac{5}{4} c_{j-1} \\ &+ D_{2,1} c_j + \frac{5}{4} c_{j+1} + 4 D_{2,2} c_{j+2} + 4\beta c_{j+3}\big) s^2 \\ &+ \big(+\beta c_{j-3} + D_{1,-2} c_{j-2} + D_{1,-1} c_{j-1} \\ &+ D_{1,1} c_{j+1} + D_{1,2} c_{j+2} - \beta c_{j+3}\big) s + c_j. \end{aligned} \tag{5}$$

where D are the coefficients: $D_{5,-1} = 11\alpha - 11\beta - \frac{5}{16}$, $D_{5,0} = 10\alpha - 10\beta - \frac{21}{16}$, $D_{5,1} = -10\alpha + 10\beta + \frac{21}{16}$, $D_{5,-2} = -11\alpha + 11\beta + \frac{5}{16}$, $D_{4,-2} = -4\alpha + 3\beta$, $D_{4,-1} = -33\alpha + 33\beta + \frac{5}{4}$, $D_{4,0} = -18\alpha + 18\beta + \frac{45}{16}$, $D_{4,1} = 32\alpha - 32\beta - \frac{15}{4}$, $D_{4,2} = 22\alpha - 22\beta - \frac{5}{16}$, $D_{4,3} = \alpha + 3\beta$, $D_{3,-2} = 6\alpha - 6\beta$, $D_{3,-1} = 28\alpha - 28\beta - \frac{15}{8}$, $D_{3,1} = -28\alpha + 28\beta + \frac{15}{8}$, $D_{3,2} = -6\alpha + 6\beta$, $D_{2,-2} = -\alpha + \beta$, $D_{2,1} = 8\alpha - 8\beta - \frac{5}{2}$, $D_{2,2} = -\alpha + \beta$, $D_{1,-2} = \alpha - \beta$, $D_{1,-1} = -6\alpha + 6\beta - \frac{5}{16}$, $D_{1,1} = 6\alpha - 6\beta + \frac{5}{16}$ and $D_{1,2} = -\alpha + \beta$.

Step 3: Assuming that function $f(x)$ has at least five continuous derivatives in interval (x_j, x_{j+1}) and by applying the Taylor theorem, the value of function in x_{j+1} is calculated. With the earlier condition on the equality of interpolation function g with function f in the k-th interpolation nodes, coefficients c from (2) are written in the form:

$$\begin{aligned}c_{j-3} = f\left(x_{j-3}\right) = \frac{27}{8}h^4 f^{(4)}\left(x_j\right) - \frac{9}{2}h^3 f^{(3)}\left(x_j\right)\\ + \frac{9}{2}h^2 f^{(2)}\left(x_j\right) - 3hf^{(1)}\left(x_j\right) + f\left(x_j\right),\end{aligned} \tag{6}$$

By continuing the procedure, the following is obtained: $c_{j-2} = f\left(x_{j-2}\right) = 2/3 \cdot h^4 f^{(4)}\left(x_j\right) - 4/3 \cdot h^3 f^{(3)}\left(x_j\right) + 2h^2 f^{(2)}\left(x_j\right) - 2hf^{(1)}\left(x_j\right) + f\left(x_j\right)$; $c_{j-1} = f\left(x_{j-1}\right) = 1/24 \cdot h^4 f^{(4)}\left(x_j\right) - 1/6 \cdot h^3 f^{(3)}\left(x_j\right) + 1/2 \cdot h^2 f^{(2)}\left(x_j\right) - hf^{(1)}\left(x_j\right) + f\left(x_j\right)$; $c_j = f\left(x_j\right)$; $c_{j+1} = f\left(x_{j+1}\right) = 1/24 \cdot h^4 f^{(4)}\left(x_j\right) + 16 \cdot h^3 f^{(3)}\left(x_j\right) + 1/2 \cdot h^2 f^{(2)}\left(x_j\right) + hf^{(1)}\left(x_j\right) + f\left(x_j\right)$; $c_{j+2} = f\left(x_{j+2}\right) = 2/3 \cdot h^4 f^{(4)}\left(x_j\right) + 4/3 \cdot h^3 f^{(3)}\left(x_j\right) + 2h^2 f^{(2)}\left(x_j\right) + 2hf^{(1)}\left(x_j\right) + f\left(x_j\right)$; $c_{j+3} = f\left(x_{j+3}\right) = 27/8 \cdot h^4 f^{(4)}\left(x_j\right) + 9/2 \cdot h^3 f^{(3)}\left(x_j\right) + 9/2 \cdot h^2 f^{(2)}\left(x_j\right) + 3hf^{(1)}\left(x_j\right) + f\left(x_j\right)$ and $c_{j+4} = f\left(x_{j+4}\right) = 32/3 \cdot h^4 f^{(4)}\left(x_j\right) + 32/3 \cdot h^3 f^{(3)}\left(x_j\right) + 8h^2 f^{(2)}\left(x_j\right) + 4hf^{(1)}\left(x_j\right) + f\left(x_j\right)$.

Step 4: By substituting it in (5), the convolution interpolation function is written in the form:

$$\begin{aligned}g\left(x\right) = a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s\\ + a_0 + \mathrm{O}\left(h^5\right),\end{aligned} \tag{7}$$

where: $a_5 = 9/8 \cdot h^2 f^{(2)}\left(x_j\right) + 11/16 \cdot h^3 f^{(3)}\left(x_j\right) + 1/4 \cdot h^4 f^{(4)}\left(x_j\right) + 9/4 \cdot hf^{(1)}\left(x_j\right) - 48\alpha hf^{(1)}\left(x_j\right) + 36\beta hf^{(1)}\left(x_j\right) - 24\alpha h^2 f^{(2)}\left(x_j\right) - 24\alpha h^3 f^{(3)}\left(x_j\right) - 10\alpha h^4 f^{(4)}\left(x_j\right) + 18\beta h^2 f^{(2)}\left(x_j\right) + 3\beta h^3 f^{(3)}\left(x_j\right)$; $a_4 = 120\alpha hf^{(1)}\left(x_j\right) - 5/4 \cdot h^3 f^{(3)}\left(x_j\right) - 5/16 \cdot h^4 f^{(4)}\left(x_j\right) - 45/8 \cdot hf^{(1)}\left(x_j\right) - 15/8 \cdot h^2 f^{(2)}\left(x_j\right) - 90\beta hf^{(1)}\left(x_j\right) + 40\alpha h^2 f^{(2)}\left(x_j\right) + 50\alpha h^3 f^{(3)}\left(x_j\right) + 46/3 \cdot \alpha h^4 f^{(4)}\left(x_j\right) - 34\beta h^2 f^{(2)}\left(x_j\right) - 2\beta h^3 f^{(3)}\left(x_j\right) - 16/3 \cdot \beta h^4 f^{(4)}\left(x_j\right)$; $a_3 = 5/8 \cdot h^3 f^{(3)}\left(x_j\right) + 15/4 \cdot hf^{(1)}\left(x_j\right) - 80\alpha hf^{(1)}\left(x_j\right) + 44\beta hf^{(1)}\left(x_j\right) - 76/3 \cdot \alpha h^3 f^{(3)}\left(x_j\right) - 86/3 \cdot \beta h^3 f^{(3)}\left(x_j\right)$ $a_2 = 5/4 \cdot h^2 f^{(2)}\left(x_j\right) + 5/48 \cdot h^4 f^{(4)}\left(x_j\right) + 24\beta hf^{(1)}\left(x_j\right) - 16\alpha h^2 f^{(2)}\left(x_j\right) - 16/3 \cdot \alpha h^4 f^{(4)}\left(x_j\right) + 16\beta h^2 f^{(2)}\left(x_j\right) + 36\beta h^3 f^{(3)}\left(x_j\right) + 16/3 \cdot \beta h^4 f^{(4)}\left(x_j\right)$ $a_1 = 5/48 \cdot h^3 f^{(3)}\left(x_j\right) + 5/8 \cdot hf^{(1)}\left(x_j\right) + 8\alpha hf^{(1)}\left(x_j\right) - 14\beta hf^{(1)}\left(x_j\right) - 2/3 \cdot \alpha h^3 f^{(3)}\left(x_j\right) - 25/3 \cdot \beta h^3 f^{(3)}\left(x_j\right)$; and $a_0 = f(x_j)$

The expansion of function f into the Taylor series is obtained:

$$\begin{aligned}f\left(x\right) = 1/24 \cdot h^4 f^{(4)}\left(x_j\right) s^4\\ + 1/6 \cdot h^3 f^{(3)}\left(x_j\right) s^3 + 1/2 \cdot h^2 f^{(2)}\left(x_j\right) s^2\\ + hf^{(1)}\left(x_j\right) s + f\left(x_j\right) + \mathrm{O}\left(h^5\right).\end{aligned} \tag{8}$$

Step 5: The interpolation error is:

$$\begin{aligned}e = f - g = \Big(\frac{1}{4}C_{5,4}h^4 f^{(4)}\left(x_j\right) + C_{5,3}h^3 f^{(3)}\left(x_j\right)\\ + C_{5,2}h^2 f^{(2)}\left(x_j\right) + C_{5,1}hf^{(1)}\left(x_j\right)\Big) s^5\\ + \Big(C_{4,4}h^4 f^{(4)}\left(x_j\right) + C_{4,3}h^3 f^{(3)}\left(x_j\right)\\ + C_{4,2}h^2 f^{(2)}\left(x_j\right) + C_{4,1}hf^{(1)}\left(x_j\right)\Big) s^4\\ + \Big(C_{3,3}h^3 f^{(3)}\left(x_j\right) + C_{3,1}hf^{(1)}\left(x_j\right)\Big) s^3\\ + \Big(C_{2,4}h^4 f^{(4)}\left(x_j\right) + C_{2,3}h^3 f^{(3)}\left(x_j\right)\\ + C_{2,2}h^2 f^{(2)}\left(x_j\right) + C_{2,1}hf^{(1)}\left(x_j\right)\Big) s^2\\ + \Big(C_{1,3}h^3 f^{(3)}\left(x_j\right) + C_{1,1}hf^{(1)}\left(x_j\right)\Big) s,\end{aligned} \tag{9}$$

where C are the coefficients: $C_{5,4} = 40\alpha - 1$, $C_{5,3} = 24\alpha - 3\beta - \frac{11}{16}$, $C_{5,2} = 24\alpha - 18\beta - \frac{9}{8}$, $C_{5,1} = 48\alpha - 36\beta - \frac{9}{4}$, $C_{4,4} = -\frac{46}{3}\alpha + \frac{16}{3}\beta + \frac{17}{48}$, $C_{4,3} = -50\alpha + 2\beta + \frac{5}{4}$, $C_{4,2} = -40\alpha + 34\beta + \frac{15}{8}$, $C_{4,1} = -120\alpha + 90\beta + \frac{45}{8}$, $C_{3,3} = \frac{76}{3}\alpha + \frac{86}{3}\beta - \frac{11}{24}$, $C_{3,1} = 80\alpha - 44\beta - \frac{15}{4}$, $C_{2,4} = \frac{16}{3}\alpha - \frac{16}{3}\beta - \frac{5}{48}$, $C_{2,3} = -36\beta$, $C_{2,2} = 16\alpha - 16\beta - \frac{3}{4}$, $C_{2,1} = -24\beta$, $C_{1,3} = \frac{2}{3}\alpha + \frac{25}{3}\beta - \frac{5}{48}$, $C_{1,1} = -8\alpha + 14\beta + \frac{3}{8}$.

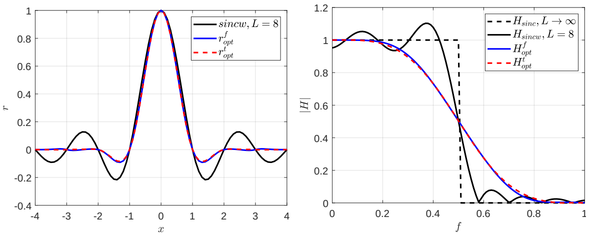
Step 6: Minimization of the interpolation error e is done by choosing appropriate kernel parameters α and β. This means that the coefficients of (9) equals zero. In this way, a system of 14 equations with two unknowns is set up. In that case, it is not possible to find a unique solution, and therefore LSM is applied. By applying it, the optimal kernel parameters are calculated: $\alpha_{opt} = 98/2707$ and $\beta_{opt} = -113/8934$.

Figure 1.a shows the time forms of: *a*) ideal interpolation kernel $sincw$. Its length is $L = 8$, windowed using a rectangular window on segment (-4, 4); *b*) polynomial fifth-order 2P kernel r_{opt}^f optimized in the spectral-domain [27] with the criterion of minimizing the ripple of the spectral characteristics ($\alpha_{opt} = 171/1408$, $\beta_{opt} = 525/7744$), and *c*) polynomial fifth-order 2P kernel r_{opt}^t optimized in the time-domain (Section 3), ($\alpha_{opt} = 98/2707$, $\beta_{opt} = -113/8934$). Figure 1.b shows the spectral characteristics: *a*) ideal interpolation kernel $sinc$, H_{sinc} ($L \longrightarrow \infty$), *b*) windowized ideal kernel H_{sincw}, ($L = 8$), *c*) 2P kernel H_{opt}^f optimized in the spectral-domain and *d*) 2P kernel H_{opt}^t optimized in the time-domain.

4 Analysis of experimental results

4.1 Experiment

An experiment is performed to enable a comparative analysis of the precision of the parametric convolution interpolation using implemented polynomial fifth-order kernels: *a*) the 1P kernel optimized is in the spectral-domain ($\alpha_{opt} = 3/64$) [24], *b*) the 2P kernel opti-



a) b)

Figure 1. a) Time forms of: ideal interpolation kernel $sincw$, polynomial fifth-order 2P kernel r^f_{opt} optimized in the spectral-domain, and polynomial fifth-order 2P kernel r^t_{opt} optimized in the time-domain; and b) Spectral characteristics of: ideal interpolation kernel H_{sinc}, windowized ideal kernel H_{sincw}, 2P kernel H^f_{opt} optimized in the spectral-domain, and 2P kernel H^t_{opt} optimized in the time-domain.

mized is in the spectral-domain ($\alpha_{opt} = 171/1408$, $\beta_{opt} = 525/7744$) [27], and c) the 2P kernel optimized in the time-domain and described in Section 3 ($\alpha_{opt} = 98/2707$, $\beta_{opt} = -113/8934 = -0.01264$). As a measure of the interpolation precision, MSE is used.

The experiment is carried out in the following steps: *Step 1*: With the interpolation kernel r of length L, the convolution interpolation of test signal f on segment (K_L, K_H) is performed. The test signals are sampled in the N interpolation nodes with sampling period h, where $N = (K_H - K_L - L)/h$.
Step 2: Test signals interpolated in interpolation points k with period Δx. The interpolation is performed in the $K = (K_H - K_L - L)/\Delta x$ points (2).
Step 3: In each interpolation point k, interpolation error $e_k = f_k - g_k$ (9) is calculated.
Step 4: MSE is defined as $\text{MSE} = \frac{1}{K}\sum_{n=1}^{K} e_k^2$ for each test signal is determined. *Step 5*: Comparative analysis of the results is performed.

Steps 1 - 5 are realized with Matlab, and except for testing, they are not intended for real time systems. Therefore, the execution time is not of a dominant importance. However, in *Step 2* the convolutional interpolation is performed, so it is important to determine the execution time of interpolation t_{exe}. For this purpose, the interpolation execution times are measured using Matlab functions *tic* and *toc*. The measurement is performed on the test platform: computer DESKTOP - S2AC43P, Processor: Intel (R) Pentium (R), CPU: G3220 3 GHZ, RAM: 8 GB, Windows 10 operating system.

To realize the interpolation within the experiment, a database of the test signals is created. It consists of test signals generated on the basis of mathematical functions, test signals with superimposed WGN, audio music signals and speech signals.

The experiment is carried out with parameters $K_L = 0$, $K_H = 35$, $h = 1$, $\Delta x = 0.01$. The results are presented using graphs and tables.

4.2 Database

The test signal database is created with:

a) Math test signals (f) formed by functions whose mathematical forms are: $f_1(x) = 1.5 \cdot \sin(x/(2\pi)) + \sin(x^2/\pi)$, $f_2(x) = 10^{-3} \cdot (x-10)(x-15)(x-35)\sin(x/\pi)$, $f_3(x) = e^{-x/2\pi} \cdot \sin(4x/\pi)$ and $f_4(x) = \sin(x/3\pi) \cdot \sin(2/\pi x)$. Math test signal f_1 is shown in Figure 3.a.

b) noise Math test signals (f_n) created by superimposing WGN with Math test signals: $f_n = f + k \cdot \text{WGN}$. By changing the value of parameter k, the noise Math test signals with SNR = 10 - 70 dB are formed. In Figure 4.a, noise Math signal f_{1n} with SNR = 10 dB is shown.

c) Music test signals (s_G) are the G tones (G1 f_0 = 48.999 Hz, G2 f_0 = 97.999 Hz, G3 f_0 = 196 Hz, G4 f_0 = 392 Hz, G5 f_0 = 783.99 Hz, G6 f_0 = 1560 Hz, G7 f_0 = 3136 Hz) which are interpreted on a Steinway B piano, the world-renowned piano manufacturer Steinway & Sons. The recording is made at the University of Iowa and is part of the RWC Music Database [28]. Music test signals are sampled with: a) f_s = 44.1 kHz, b) f_s = 22.05 kHz and c) f_s = 8 kHz with 16 bps. Figure 5 shows the time form of the Music test signal s_{G3}: a) tone duration T = 3.8 s (Figure 5.a) and b) frame T = 32 ms (Figure 5.b).

d) The speech test signals (s) are taken from the EMODB database of the German Emotional Speech which was created by the Institute of Communication Science, Technical University, Berlin, Germany [29]. For the experiment, the precision of the interpolation is tested with the speech test signals which are in the EMODB database named: 03a01Fa, 08a01Ab, 09a02Wb, 10b09Ad, 11a02Wc, 12a01Fb, 13a04Ac, 14b02Na, 15b09Ac, 16b10Td. The data is recorded at a f_s = 48 kHz sampling rate and then down-sampled to f_s = 16 kHz and f_s = 8 kHz. Figure 6 shows the time form of speech test signal $s_{08a01Ab}$: a) duration T = 1.8 s (Figure 6.a) and b) frame T = 32 ms (Figure 6.b).

4.3 Results

The mean-square error, which refers to the estimation of the interpolation error of Math test function $f_2(x)$, for $\beta = 0$, depending on kernel parameter α, is shown in Figure 2.a, for: a) the theoretical estimation of MSE_{est} (9), and b) the experimental estimation of MSE_{exp} (by the convolution interpolation) (2). By locating the minimum of MSEs, the optimal values of parameters α are determined: $\text{MSE}_{est\_min} = 2.2765 \cdot 10^{-7}$, $\alpha_{est\_opt} = 0.0490$, $\text{MSE}_{exp\_min} = 4.5567 \cdot 10^{-7}$, $\alpha_{exp\_opt} = 0.0490$. Figure 2.b shows the theoretical dependence of MSE_{est} on kernel parameters α and β. By locating the minimum of MSE_{est}, the optimal values of parameters α and β are determined: $\text{MSE}_{est\_min} = 1.0816 \cdot 10^{-8}$, $\alpha_{est\_opt} = 0.0610$ and $\beta_{est\_opt} = 0.0100$. Figure 2.c shows the experimental dependence of MSE_{exp} on

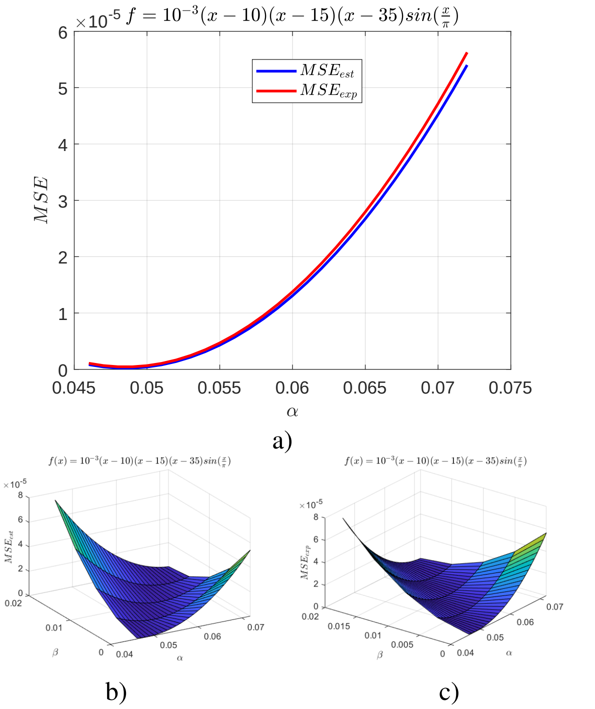


Figure 2. a) MSE depending on parameter α, for $\beta = 0$ estimated theoretically (MSE$_{est}$) and experimentally (MSE$_{exp}$); b) MSE depending on kernel parameters α and β, estimated theoretically (MSE$_{est}$); and c) MSE depending on kernel parameters α and β determined by experiment (MSE$_{exp}$).

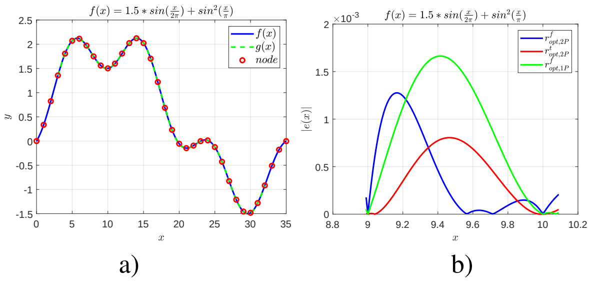


Figure 3. Interpolated signal f, interpolation function g and interpolation nodes for test function a) f_1. Absolute interpolation error e on segment (9, 10) for b) f_1.

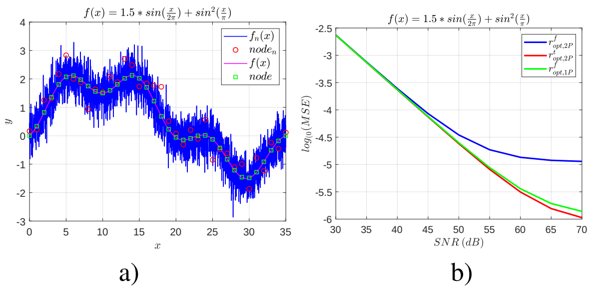


Figure 4. a) Interpolated Math test signals $f1$ and noise Math test signal $f1n$ with SNR = 10 dB. b) interpolation error log_{10}(MSE(SNR)) when interpolating noise Math test signal $f1n$ with SNR = 30 – 70 dB using tested interpolation kernels.

kernel parameters α and β. By locating the minimum of MSE$_{exp}$, the optimal values of the parameters α and β are determined: MSE$_{exp_min}$ $= 4.0326 \cdot 10^{-7}$, $\alpha_{exp\_opt} = 0.0550$ and $\beta_{exp\_opt} = 0.0050$.

The time forms of Math test function f_1, interpolation function g, and interpolation nodes are shown in Figure 3.a. Absolute interpolation error $|e| = |f - g|$, for tested optimized kernel parameters ($r^f_{opt,1P}$, $r^f_{opt,2P}$ and $r^t_{opt,2P}$), on segment (9, 10), for test function f_1 is shown in Figure 3.b.

The mean value ($\overline{\text{MSE}}_f$) of minimum MSE for all analyzed optimized kernel parameters ($r^f_{opt,1P}$, $r^f_{opt,2P}$ and $r^t_{opt,2P}$), for all Math test functions is shown in Table 1.

Mean value $\overline{\text{MSE}}_{f,SNR}$ of the minimum MSE for all analyzed optimized kernel parameters, for all noise Math test signals with SNR = 30 – 70 dB is shown in Table 1. The minimum interpolation error depending on SNR, when interpolating Math test signal f_{1n}, is shown in Figure 4.b.

Mean values of minimum MSE for all analyzed optimized kernel parameters, for all tested music test signals (tones s_{G1} - s_{G7}), for sampling frequencies: a) f_s = 44.1 kHz ($\overline{\text{MSE}}_{G,44kHz}$), b) f_s = 22.05 kHz ($\overline{\text{MSE}}_{G,22kHz}$) and c) f_s = 8 kHz ($\overline{\text{MSE}}_{G,8kHz}$), are shown in Table 1.

The Mean values of the minimum MSE for all analyzed optimized kernel parameters, for all tested speech test signals which were sampled with a) f_s = 16 kHz ($\overline{\text{MSE}}_{S,16kHz}$) and b) f_s = 8 kHz ($\overline{\text{MSE}}_{S,8kHz}$) are shown in the Table 1.

Table 1 shows the averaged interpolation errors MSE$_{sum}$ for: a) MSE$_f$ (Math test signals); b) MSE$_{f_n}$, (noise Math test signasl); c) MSE$_{G,44kHz}$, MSE$_{G,22kHz}$ and MSE$_{G,8kHz}$ (Music test signals sampled with f_s = 44.1 kHz, f_s = 22.05 kHz and f_s = 8 kHz); and d) MSE$_{s,16kHz}$ and MSE$_{s,8kHz}$ (the speech test signals sampled with f_s = 16 kHz and f_s = 8 kHz).

The execution time of the convolution interpolation (Step 2) $t_{exe} = 9.2341 \cdot 10^{-8}$ s, as the arithmetic mean of the execution time for 100000 interpolations is determined.

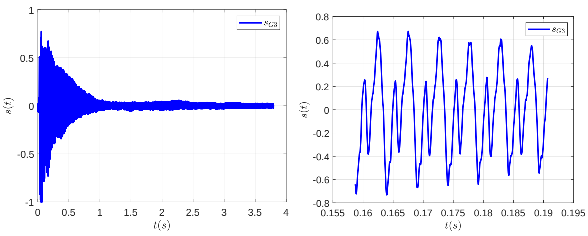
4.4 Analysis of the results

Based on the results shown in Table 1, refering to the interpolation of Math test signals *f*, it is concluded that the precision of the convolution interpolation using polynomial fifth-order 2P kernel $r^t_{opt,2P}$, whose optimal parameters are determined by optimization in the time-domain (Section 3), is higher compared to the interpolation which is used:

a) the 1P fifth-order kernel, whose optimal parameters are determined in the spectral-domain [24] $\overline{\text{MSE}}_f, (r^f_{opt,1P})$ / $\overline{\text{MSE}}_f, (r^t_{opt,2P})$ = $1.5556 \cdot 10^{-6}/8.4974 \cdot 10^{-7}$ = 1.8306 times, and

Table 1. Averaged interpolation errors MSE_{sum}.

	$MSE^f_{min,1P}$ $(r^f_{opt,1P})$ $(\times 10^{-6})$	$MSE^f_{min,2P}$ $(r^f_{opt,2P})$ $(\times 10^{-6})$	$MSE^t_{min,2P}$ $(r^t_{opt,2P})$ $(\times 10^{-6})$
Math Test functions: f_1, f_2, f_3, f_4			
$\overline{MSE_f}$	1.5556	8.2445	0.84974
Noise functions: f_{1n}, ..., f_{4n},			
$\overline{MSE_{f,SNR}}$	156.02	163.27	143.10
Music signal (G1-G7)			
$\overline{MSE_{G,44kHz}}$	16.123	15.268	14.810
$\overline{MSE_{G,22kHz}}$	242.74	235.89	238.92
$\overline{MSE_{G,8kHz}}$	4183.3	4102.4	4042.9
Speech signal			
$\overline{MSE_{S,16kHz}}$	1192.8	1174.2	1164.9
$\overline{MSE_{S,8kHz}}$	3351.9	3090.4	3068.0
Averaged MSE			
$\overline{MSE_{sum}}$	1306.3	1255.6	1239.1



a) b)

Figure 5. Time form of music test signal s_{G3}: a) tone duration T = 3.8 s and b) frame T = 32 ms

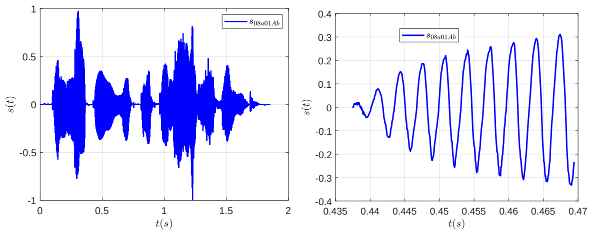
b) the 2P fifth-order kernel, whose optimal parameters are determined in the spectral-domain [27] $\overline{MSE_f}, (r^f_{opt,2P})$ / $\overline{MSE_f}, (r^t_{opt,2P})$ = $8.2445 \cdot 10^{-6} / 8.4974 \cdot 10^{-7}$ = 9.7023 times.

In the interpolating of the noise Math signals f_n (SNR = 10 – 70 dB), (Table 1) using the interpolation kernel $r^t_{opt,2P}$, the precision of the convolution interpolation is higher compared to the interpolation using:

a) the 1P kernel: $\overline{MSE_{f,SNR}}, (r^f_{opt,1P})$ / $\overline{MSE_{f,SNR}}, (r^t_{opt,2P})$ = $1.5602 \cdot 10^{-4}$ / $1.4310 \cdot 10^{-4}$ = 1.0902 times, and

b) the 2P kernel: $\overline{MSE_{f,SNR}}, (r^f_{opt,2P})$ / $\overline{MSE_{f,SNR}}, (r^t_{opt,2P})$ = $1.6327 \cdot 10^{-4}$ / $1.4310 \cdot 10^{-4}$ = 1.1409 times.

The interpolation precision when applying kernel $r^t_{opt,2P}$ is higher compared to kernels $r^f_{opt,1P}$ and $r^f_{opt,2P}$ used in interpolating Music test signals s_G (tones G1 - G7), which are sampled with:



a) b)

Figure 6. Time form of the speech test signal $s_{08a01Ab}$: a) duration T = 1.8 s and b) frame T = 32 ms.

a) f_s = 44.1 kHz: $\overline{MSE_{G,44kHz}}, (r^f_{opt,1P})$ / $\overline{MSE_{G,44kHz}}, (r^t_{opt,2P})$ = $1.6123 \cdot 10^{-5}$ / $1.4810 \cdot 10^{-5}$ = 1.0886 times and $\overline{MSE_{G,44kHz}}, (r^f_{opt,2P})$ / $\overline{MSE_{G,44kHz}}, (r^t_{opt,2P})$ = $1.5268 \cdot 10^{-5}$ / $1.4810 \cdot 10^{-5}$ = 1.0309 times;

b) f_s = 22.05 kHz: $\overline{MSE_{G,22kHz}}, (r^f_{opt,1P})$ / $\overline{MSE_{G,22kHz}}, (r^t_{opt,2P})$ = $2.4274 \cdot 10^{-4}$ / $2.3892 \cdot 10^{-4}$ = 1.0159 times and $\overline{MSE_{G,22kHz}}, (r^f_{opt,2P})$ / $\overline{MSE_{G,22kHz}}, (r^t_{opt,2P})$ = $2.3589 \cdot 10^{-4}$ / $2.3892 \cdot 10^{-4}$ = 0.9873 times; and

c) f_s = 8 kHz: $\overline{MSE_{G,8kHz}}, (r^f_{opt,1P})$ / $\overline{MSE_{G,8kHz}}, (r^t_{opt,2P})$ = $4.1833 \cdot 10^{-4}$ / $4.0429 \cdot 10^{-4}$ = 1.0347 times and $\overline{MSE_{G,8kHz}}, (r^f_{opt,2P})$ / $\overline{MSE_{G,8kHz}}, (r^t_{opt,2P})$ = $4.1024 \cdot 10^{-4}$ / $4.0429 \cdot 10^{-4}$ = 1.0147 times.

The interpolation precision when applying kernel $r^t_{opt,2P}$ is higher compared to kernels $r^f_{opt,1P}$ and $r^f_{opt,2P}$ used in interpolating the speech signals s, sampled with:

a) f_s = 16 kHz: $\overline{MSE_{S,16kHz}}, (r^f_{opt,1P})$ / $\overline{MSE_{S,16kHz}}, (r^t_{opt,2P})$ = $1.1928 \cdot 10^{-3}$ / $1.1649 \cdot 10^{-3}$ = 1.0239 times and $\overline{MSE_{S,16kHz}}, (r^f_{opt,2P})$ / $\overline{MSE_{S,16kHz}}, (r^t_{opt,2P})$ = $1.1742 \cdot 10^{-3}$ / $1.1649 \cdot 10^{-3}$ = 1.0079 times; and

b) f_s = 8 kHz (Table 1): $\overline{MSE_{S,8kHz}}, (r^f_{opt,1P})$ / $\overline{MSE_{S,8kHz}}, (r^t_{opt,2P})$ = $3.3519 \cdot 10^{-4}$ / $3.0680 \cdot 10^{-4}$ = 1.0925 times and $\overline{MSE_{S,8kHz}}, (r^f_{opt,2P})$ / $\overline{MSE_{S,8kHz}}, (r^t_{opt,2P})$ = $3.0963 \cdot 10^{-4}$ / $3.0680 \cdot 10^{-4}$ = 1.009 times.

Considering the precision of the convolution interpolation of each analyzed test signal, by calculating average MSE, it is concluded that the precision of the interpolation using fifth-order 2P kernel $r^t_{opt,2P}$, whose optimization is performed in the time-domain (Section 3), compared to the application of kernels whose optimization is performed in the spectral-domain, is higher:

a) the 1P kernel: $\overline{MSE_{sum}}, (r^f_{opt,1P})$ / $\overline{MSE_{sum}}, (r^t_{opt,2P})$ = $1.3063 \cdot 10^{-3}$ / $1.2391 \cdot 10^{-3}$ = 1.0542 times, and

b) the 2P kernel: $\overline{MSE_{sum}}, (r^f_{opt,2P})$ /

$\overline{\mathrm{MSE}_{sum}, (r^t_{opt,2P})}$ = $1.2556 \cdot 10^{-3}$ / $1.2391 \cdot 10^{-3}$ = 1.0133 times.

According to the theoretical analysis of interpolation error e (9) of 2P kernel $r^t_{opt,2P}$, which is realized in the time-domain, as well as the experimental results and the comparative analysis of the results, which refer to the interpolation precision, in relation to the results obtained using the 1P and 2P kernels optimized in the spectral-domain, it is confirmed that the choice of the optimal parameter values is appropriate.

The execution time of the interpolation with interpolation kernel $r^t_{opt,2P}$ implemented with Matlab, is $t_{exe} = 9.2341 \cdot 10^{-8}$ s. However, for the real-time interpolation, the convolution algorithm must be written in a programming language (for example, programming language C) where, in the compilation process, optimizations reduces the program execution time.

Taking into account: a) the previously presented comparative analysis of the interpolation precision, and b) the execution time of the interpolation, the 2P fifth-order interpolation kernel $r^t_{opt,2P}$ with the parameters $\alpha_{opt} = 98/2707$ and $\beta_{opt} = -113/8934$, can be recommended for the implementation in the real-time systems.

5 Conclusion

The paper describes optimization process of the polynomial fifth-order two-parameter interpolation kernel. The optimization of the 2P kernel involves a selection of the optimal value of kernel parameters α_{opt} and β_{opt}. The optimization is realized by minimizing interpolation error e in the time-domain. First, fifth-order 2P kernel r defined on interval (-4, 4) is described. Then, by applying the convolution interpolation between interpolated function f with 2P kernel r, interpolation function g is determined. Interpolated functions f and g in the interpolation nodes are equal. Interpolation error e in interval $x_j \leqslant x \leqslant x_{j+1}$, is then determined. Provided function f has at least five continuous derivatives in the interval (x_j, x_{j+1}), interpolation error e is developed in the Taylor series up to the fifth term. According to the minimization criterion, it is necessary to agree well functions f and g, up to the fifth term. By minimizing the first five terms of the Taylor series of interpolation error e, the optimal value of the kernel parameter can be calculated. However, the minimization process does not lead to a unique solution for α and β. Each of the five terms of the Taylor series has a coefficient. It depends on several members represented in the form $(a \cdot \alpha + b \cdot \beta + c)$. A system of 14 equations with two unknowns is formed. Such a system of equations has no unique solution. Using LSM, the optimal values of the kernel parameters are determined: $\alpha_{opt} = 98/2707$ and $\beta_{opt} = -113/8934$. By using an experiment, the proposed optimal value of the 2P kernel parameter is verified. A database of the test signals is created for the experiment. It is composed of the test signals of four groups: the Math test signal, noise Math signal, musuc test signal and speech test signal. Each test function is interpolated by the convolution interpolation, using the interpolation kernels of fifth-order 1P and 2P, optimized in the spectral-domain. For each interpolation, errors e are calculated, and, based on them, MSEs, which are used for a comparative analysis, are formed. The analysis shows that the interpolation precision of the 2P kernel optimised in the time-domain is the highest. In this way, the suggested optimal values of the kernels parameters are verified.

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