ELEKTROTEHNIŠKI VESTNIK 88(5): 285-289, 2021
ORIGINAL SCIENTIFIC PAPER

# Mathematical Models to Study Series Compensation of the Induction-Motor Reactive Power

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**Abstract.** The paper proposes mathematical models and algorithms to calculate processes and characteristics of induction motors in a series compensation of the reactive power. The developed algorithms are based on a mathematical model of the induction motor in phase coordinates. A complete system of differential equations of the electromechanical equilibrium of the stator and rotor circuits is used for a transient analysis. The algorithm to calculate the static characteristics is based on the projection method for solving the boundary value problem for a system of equations of the electrical equilibrium.

**Keywords:** induction motor, reactive power, series compensation, boundary problem, static characteristics

**Matematični modeli za analizo zaporedne kompenzacije jalove moči indukcijskega motorja**

V prispevku predlagamo matematične modele in algoritme za izračun procesov in značilnosti asinhronih motorjev pri zaporedni kompenzaciji jalove moči. Razviti algoritmi temeljijo na matematičnem modelu asinhronega motorja v faznih koordinatah. Za analizo prehoda je uporabljen sistem diferencialnih enačb elektromehanskega ravnotežja statorskih in rotorskih tokokrogov. Algoritem za izračun statičnih lastnosti temelji na projekcijski metodi za reševanje mejnega problema za sistem enačb električnega ravnotežja.

## 1 INTRODUCTION

Induction motors (IM) are consumers of the reactive power whose transmission from the generator to the place of consumption causes an additional loading of the power line with reactive currents [1, 2]. The problem of reactive-power compensation is always relevant. It must be solved separately for each electric drive to avoid worsening the economic performance of the power supply system as a whole. One way to compensate for the reactive power is to install capacitor devices in the power distribution networks connected in parallel to the consumers or series. Such compensation can be of a group or individual type [3, 4]. The peculiarity of the series compensation is that the load current flows through the capacitors, which at any time can vary widely. Hence, the reactive power of the series-connected capacitors is also a variable. The capacitors capacitance must be continuously adjusted to maintain the power factor (*cos* φ) at a given level [5]. Such control can be either manual or automatic [6], but for the microprocessor control, programs for calculating processes in real-time are needed.

Most electric drives use direct IM starting with a squirrel-cage rotor, at which the starting current can reach eight times its value. This current value is not harmful to the motor, but in the case of prolonged and frequent starts, the temperature of the motor windings may exceed the permissible limits. Moreover, the starting current can significantly reduce the mains voltage or impair or completely disrupt other consumers' regular operation when starting powerful motors. Therefore, there is a need to research the start-up process of the electric drive system on a mathematical model [5–7] to develop a control system for the start-up process to reduce the start-up time or the current value.

The choice of the capacitance capacity based on a simplified IM mathematical model can be unreliable. Its inadequacy can lead to an IM overloading, the appearance of self-excitation [8, 9], or non-starting. However, using a dynamic IM mathematical model of high-level adequacy requires appropriate high-efficiency calculation methods.

The coordinate system's choice is an important task in the mathematical modeling of IM processes with capacitors in the stator winding phases. In particular, phase voltages are converted to the orthogonal coordinate axes in the automatic control systems. However, their use is not always effective because it is impossible to describe processes in transformed coordinates adequately. Using a mathematical model in phase coordinates increases the accuracy of solving and expanding the problem range. The operation of the developed algorithms must be of a high-speed to allow for real-time process control.

Choosing the capacitor capacitance using a simplified IM mathematical model can be unreliable and

*Received 25 October 2021*
*Accepted 19 November 2021*

inadequate, leading to an overcurrent [12]. The paper presents a method and algorithm for mathematical modeling of a three-phase IM with series-connected capacitors developed using an IM mathematical model in phase coordinates to determine the capacitor capacitance to compensate for the reactive power according to the mode of operation.

## 2 ELECTRICAL EQUILIBRIUM EQUATION

The IM mathematical models developed according to the theory of circuits in three-phase physical coordinates enable consideration of various factors that define the behavior of the electric drive system. However, the IM electrical equilibrium differential equations (DE) in three-phase physical coordinates impacted by the rotor rotation have periodic coefficients. They significantly complicate their solution and make it almost impossible to calculate the static characteristics. The numerical analysis algorithms of the IM processes with capacitors connected in series to the stator winding are developed in fixed three-phase axes [11]. They are physical for the stator windings and fixed three-phase for the rotor phases. The electric circuit of the motor loops is shown in Fig. 1.

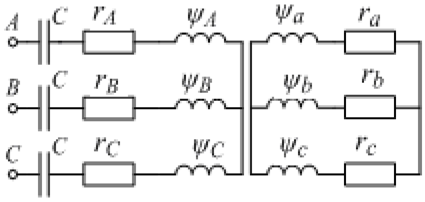



Fig. 1. IM electric circuit with the capacitor capacitance ($C$)

The system of the electrical equilibrium equations to describe the dynamic IM mode has the following form:

$$
\begin{gathered}
\frac{d\psi_A}{dt}-\frac{d\psi_B}{dt}=-r_A i_A+r_B i_B-u_{kA}+u_{kB}+u_{AB};\\
\frac{d\psi_B}{dt}-\frac{d\psi_C}{dt}=-r_B i_B+r_C i_C-u_{kB}+u_{kC}+u_{BC};\\
i_A+i_B+i_C=0;\\
\frac{d\psi_a}{dt}-\frac{d\psi_b}{dt}=-r_a i_a+r_b i_b-\omega\left(\psi_b-2\psi_c+\psi_a\right)/\sqrt{3};\\
\frac{d\psi_b}{dt}-\frac{d\psi_c}{dt}=-r_b i_b+r_c i_c-\omega\left(\psi_c-2\psi_a+\psi_b\right)/\sqrt{3};\\
i_a+i_b+i_c=0;\\
\frac{du_{kA}}{dt}=\frac{i_A}{C};\quad \frac{du_{kB}}{dt}=\frac{i_B}{C};\quad \frac{du_{kC}}{dt}=\frac{i_C}{C},
\end{gathered}
\tag{1}
$$

where $\psi_\eta$, $i_\eta$, $r_\eta$ are the flux linkage, currents, and resistances respectively of contours ($\eta = A, B, C, a, b, c$); $u_{AB}, u_{BC}$ are the corresponding line voltages; $u_{kA}, u_{kB}, u_{kC}$ are the voltages on capacitors with capacitance $C$; $\omega=\omega_0(1-s)$ is the angular speed of the rotor rotation expressed in electric radians per second; $s$ is the slip; $\omega_0$ is the mains voltage frequency.

Equation (2) describes the dynamics of the rotor motion

$$
d\omega/dt = p_0\left(M_e - M_c\right)/J, \tag{2}
$$

where $M_e$ is the IM electromagnetic torque in phase coordinates [11]; $M_c$ is the loading moment on a motor shaft; $J$ is the inertia equivalent moment of the electric drive system; $p_0$ is the number of the motor pole pairs.

From the finite system equations (1) we can move to the differential ones. The electromechanical processes are described by a DE nonlinear system consisting of equations (1), (2). Given that the flux linkage of each circuit depends on the currents, the equation of the DE system (1) written in a matrix form is

$$
\begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} \times \begin{vmatrix} d\vec{B}_1/dt \\ d\vec{B}_2/dt \\ d\vec{B}_3/dt \end{vmatrix} = \begin{vmatrix} F_1 \\ F_2 \\ F_3 \end{vmatrix}, \tag{3}
$$

where:

$$
A_{11}=\begin{vmatrix} L_{AA}-L_{BA} & L_{AB}-L_{BB} & L_{AC}-L_{BC} \\ L_{BA}-L_{CA} & L_{BB}-L_{CB} & L_{BC}-L_{CC} \\ 1 & 1 & 1 \end{vmatrix};
$$

$$
A_{12}=\begin{vmatrix} L_{Aa}-L_{Ba} & L_{Ab}-L_{Bd} & L_{Ac}-L_{Bc} \\ L_{Ba}-L_{Ca} & L_{Bb}-L_{Cb} & L_{Bc}-L_{Cc} \\ 0 & 0 & 0 \end{vmatrix};
$$

$$
A_{21}=\begin{vmatrix} L_{aA}-L_{bA} & L_{aB}-L_{bB} & L_{aC}-L_{bC} \\ L_{bA}-L_{cA} & L_{bB}-L_{cB} & L_{bC}-L_{cC} \\ 0 & 0 & 0 \end{vmatrix};
$$

$$
A_{22}=\begin{vmatrix} L_{aa}-L_{ba} & L_{ab}-L_{bb} & L_{ac}-L_{bc} \\ L_{ba}-L_{ca} & L_{bd}-L_{cb} & L_{bc}-L_{cc} \\ 1 & 1 & 1 \end{vmatrix};
$$

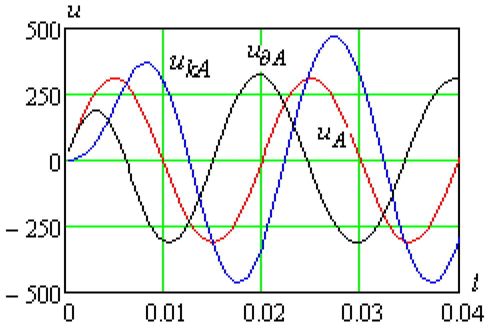
$$
A_{13}=0;\quad A_{23}=0;\quad A_{31}=0;\quad A_{32}=0;
$$

$$
A_{33}=diag(1,1,1)
$$

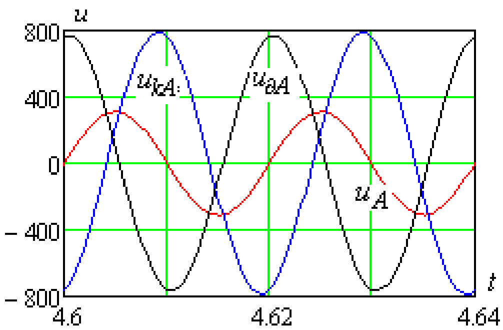
$$
B_1=\begin{vmatrix} di_A/dt \\ di_B/dt \\ di_C/dt \end{vmatrix};\ B_2=\begin{vmatrix} di_a/dt \\ di_b/dt \\ di_c/dt \end{vmatrix};\ B_3=\begin{vmatrix} du_{kA}/dt \\ du_{kB}/dt \\ du_{kC}/dt \end{vmatrix}
$$

$$
\begin{gathered}
F_1 = u_{AB} - r_A i_A + r_B i_B - u_{kA} + u_{kB};\\
F_2 = u_{BC} - r_B i_B + r_C i_C - u_{kB} + u_{kC};\\
F_3 = 0.
\end{gathered}
$$

Its numerical integration, makes it possible to calculate the time dependence of the circuit current and rotor speed and research the impact of each coordinate (capacitor capacitance, moment of inertia, load moment) of the system on the transient process. Examples of the time dependence of the stator phase currents and rotor speed during a motor start-up are 4A80B2Y3 ($P$=2.2 kW and $U$ = 220 V, $I$ = 4.7 A) respectively.




a)




b)

Fig. 2. Dependence IM voltage-time at no-load state with the capacitors with the 300 μF in series connected: supply of stator phase A ( $u_A$ ), capacitor ( $u_{kA}$ ) and on the motor ( $u_{\partial A}$ ) during start-up (*a*) and upon a transient process (*b*)

Fig. 2 shows that the motor voltage exceeds the supply voltage during the start-up process, which reduces the start-up time. However, when there is resonance at some capacitance value during the start-up process (Fig. 3), the voltage and current considerably exceed their nominal values.

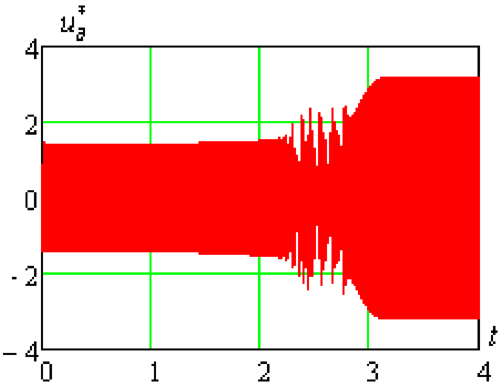



Fig. 3. Time dependence of the motor relative, voltage value during motor the start-up at its rated load and 300 $\mu F$ capacitors capacitance.

To avoid the IM voltage and current overload following the start-up, the capacitors capacitance should be reduced. Fig. 4 shows an example of a calculated transient process occurring during an IM start-up where the 300 μF capacitor capacitance reduces to 100 μF is

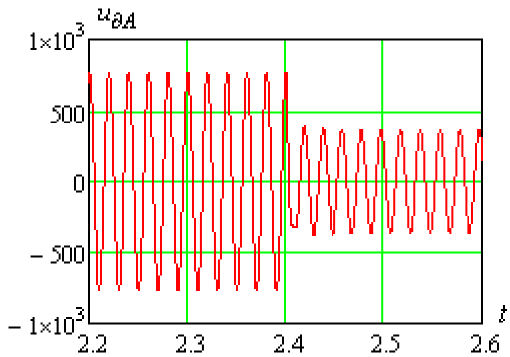



Fig. 4. IM dependence on the voltage-time start-up of the rated load

## 3 Static Characteristics

Though the IM start-up process is transient, analysis can be performed using its static characteristics. They are a sequence of steady-state modes of the IM operation at different slips. If the slip is constant, due to a periodic perturbation (supply voltage), the system DE (1) describes a stationary mode. In this case, the relationship of fluxes and currents and their dependent coordinates change according to a periodic law with period $T = 1/f$, where $f$ is the frequency of the supply voltage. The calculation of the periodic mode is a boundary value problem for a system of nonlinear DE (1) of the first order with periodic boundary conditions. The result of its solution is the dependences coordinates during period $T$.

Consider the algorithm to calculate the static characteristics is obtained by writing system DE (1) of the electrical equilibrium of the IM contours in the form of a vector equation

$$\frac{d\,\vec{y}(\vec{x},t)}{dt} = \vec{z}(\vec{y},\vec{x},t) + \vec{u}(t), \qquad (4)$$

where:

$$\vec{y} = (\vec{\psi},\vec{u}_k)^*;\ \vec{x} = (\vec{i},\vec{u}_k)^*;\ \vec{u}(t)$$

vector periodic functions of time in which:

$$\vec{\psi} = (\psi_A,\psi_B,\psi_C,\psi_a,\psi_b,\psi_c)^*;\ \ \vec{u}_k = (u_{kA},u_{kB},u_{kC})^*;$$

$$\vec{i} = (i_A,i_B,i_C,i_a,i_b,i_c)^*;\ \ \vec{u}(t) = \left(u_{AB}(t),\, u_{BC}(t),\, 0,\ldots,0\right)^*$$

Is a given vector function of a periodic perturbation.

Since the system of equations (4) includes time coordinate $t$, each slip value $s$ corresponds to the periodic dependence of:

$$\vec{x}(t) = \vec{x}(t+T),$$

which describes the change in the coordinates during a period. Thus, the answer to the boundary-value problem is not the determination of the coordinates at the one-time point but the determination of the coordinates' functional dependences at period *T*.

The simplest method to determine the dependences is a numerical integration of equation (4) to the steady-state process. However, using the method, the static characteristics cannot be calculated. Assuming the problem in boundary, the coordinates periodic functional dependences can be obtained without resorting to the analysis of the transient process.

Forming a boundary value problem requires algebraizing the DE system (1) over a period, namely, creating its discrete analog. The presented calculation algorithms are based on the DE algebraization (4) using cubic splines' approximation of the state variables [9]. Their properties allow performing the calculation with a minimum amount solving of calculations. Solving the system allows finding an approximate solution in numerical values of state variables on a grid of nodes of period *T*.

Calculating the capacitors capacitance to compensate for the reactive power and the laws of its control in specific modes is made by calculating the multidimensional static characteristic as the dependence of the set of nodal coordinates of the mode on the value of the capacitor capacitance for each slip *s*.

## 4 ALGORITHM TO SOLVE THE BOUNDARY VALUE PROBLEM

With a spline approximation of coordinates on the grid of *N* + 1 nodes of the period by splines of the third-order [10], a discrete analog of the DE system (4) of the *m*-th order in the form of a nonlinear algebraic equation of order Nm is obtained

$$H\vec{Y} + B\vec{Z} = -B\vec{U}\ , \qquad (4)$$

where *H*, *B* are block-diagonal matrices of the spline coefficients, their elements are determined by a grid of nodes;

$$\vec{Y} = \left(\overline{\vec{y}_1, \vec{y}_N}\right)^*; \vec{Z} = \left(\overline{\vec{z}_1, \vec{z}_N}\right)^*; \vec{U} = \left(\overline{\vec{u}_1, \vec{u}_N}\right)^*$$

are vectors. Each of them consists of *N* vectors of nodal values of the corresponding variables. In system (4), there is an unknown vector $\vec{X} = (\overline{\vec{x}_1, \vec{x}_N})^*$.

System (4) is solved by a continuation method with an iterative refinement by the Newton method. The elements of the Jacobian matrix are resistances, capacitors capacitances and differential inductances of the IM circuits. They are determined for each j-th node of the grid $j = \left(\overline{1, N}\right)$ according to [11].

The coordinates research depends on the capacitor capacitance for a given slip value at a constant voltage obtained using the differential method [10]. This is made by we differentiating the algebraic equation (4) by *C*. As a result, the DE system of argument *C* is obtained. Its Jacobian matrix is the same as in (6), and the right-hand sides are vector $\partial\vec{Z} / \partial C$.

For example, fig. 5 shows the static characteristic calculation results in dependences on the capacitors capacitance, RMS voltage on the stator phase A capacitor, and the relative value of the electromagnetic torque 4A80B2Y3 motor during a slip of $s = 1,0$. As seen from the figure, the curves clearly express the maxima corresponding to different capacitor capacitance values.

The calculated dependence of the current and active and reactive power in relative units on the capacitors capacitance is shown in Fig. 6.

When the capacitors capacitance is 600 $\mu F$ there is a total compensation of the IM reactive power. In case of its increase, there is an overcompensation (the reactive power changes it positively to negative).

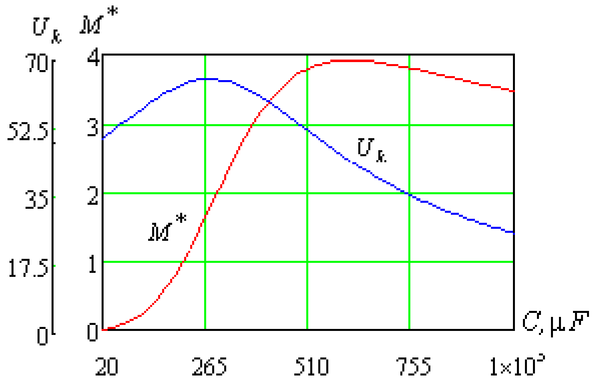



Fig. 5. Dependence oa the capacitors capacitance RMS voltage on capacitors ($U_k$) and the relative value of the electromagnetic torque ( $M^*$ ) at slip $s = 1.0$ .

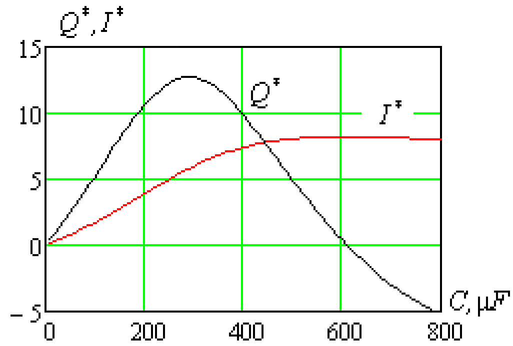



Fig. 6. Dependence of the relative values of current ( $I^*$ ) and reactive powers ( $Q^*$ ) on the capacitors capacitance (*C*)

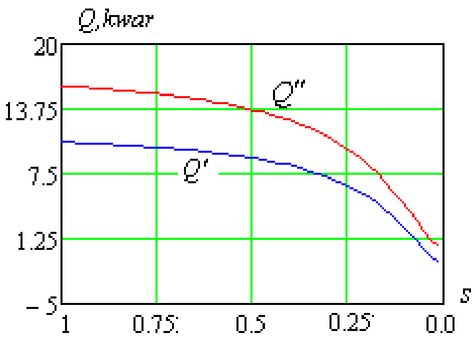



Fig. 7. Reactive power dependence on the slip of at the input of the electric drive system ($Q'$) and the motor input ($Q''$) within series-connected capacitors $C=300\ \mu F$

The dependence shown in Fig. 7 shows how much of the reactive power consumed by the electric drive system changes due to the in-series connected capacitors to IM.

## 5 Conclusion

1. Mathematical models are proposed to be used in for numerical analysis of dynamic modes and static characteristics of IM with a series compensation of the reactive power. They are based on the system of differential equations of the electromechanical equilibrium written in three-phase fixed coordinates. They are used to calculate algorithms and develop computer models to determine capacitance impact on the IM start-up dynamics and static characteristics.

2. Static characteristics are calculated using a projection method that enables solving the boundary value problem for the DE system of the electric equilibrium of the IM circuits, essential functions of the cubic splines, and method of parametric differentiation. The calculations show a periodic dependence on the coordinates. Using the projection method, they are obtained in a timeless domain. The parameter differentiation method determines the impact of variation in any parameter on the nature of the variation in coordinates during the period.

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