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A Simple Method to be Used in Calculation and Analysis of a Rectangular Cross-section Short Cylindrical Coils Inductance

Miloje Kostić

Electrical Engeneering Institute Nikola Tesla, Belgrade, SERBIA
E-male: mkostic@ieent.org

Abstract: In this paper, a simplified procedure is presented to be used in determination of magnetic reluctance and inductance of short cylindrical coils of a rectangular cross- section. Based on calculations and analysis, it is shown that the coil with a greater ratio of longitudinal/transversal dimensions of the cross-section should rather be approximated with the rectangular cross-section than the circular cross-section, as it is usually done. It is proven that the magnetic conductance to the flux outside the inner part of the coil is directly proportional to the perimeter of the contour of the cross-section given. By using obtained results, the impact of the cross-section shape on the values of the magnetic reluctance and inductance of short cylindrical coils can be analyzed.

Key-Words: Inductance, Reluctance, Short coil, Rectangular cross-section.

1 Introduction

Self-inductance is a fundamental electrical engineering parameter for a coil. Several monographs are devoted to its calculation [1], [2]. Based on the definition of flux linkage per ampere, self-inductance can be computed by applying the Biot-Savart law directly. Alternate methods include 1) integration of the mutual inductance of two circuit elements over the cross-section of the conductor and 2) Taylor's series expansion [1], [2].

It is difficult to assure the inductance calculation to be accurate. Traditionally, two choices have been available. One way is by double interpolation from tables in a monograph [1], [2]. The other way is to estimate its value from an approximate formula and corresponding coefficient values, given in the form of tables or graphics in a literature [3]-[7].

In this paper, accurate self-inductance expressions based on further similarity of expressions for the value of reluctance to the flux outside solenoid interior of circular and rectangular cross-section. On this basis, the corresponding expression would be established enabling calculation of rectangular cross-section solenoid total reluctance (R_m). It would be especially usefully for the analysis of the impact of the cross-section shape on the values of magnetic reluctance and inductance of short cylindrical coils.

On the basis the known approximations as the expression for nductance (L) and reluctance (R_m) of cylindrical coils of circular cross-section, [3] and [4]:

$$L_0 = \mu_0 \frac{D^2 \pi}{\ell + 0.45 D} \tag{1}$$

$$R_{m0} = \frac{1}{\mu_0} \frac{\ell + 0.45 D}{D^2 \pi / 4} \tag{2}$$

Equation (3), for the magnetic reluctance to flux outside interior of short coil, is derived [5]

$$\Delta R_{m0} = \frac{1}{\mu_0} \cdot \frac{0.45 D}{D^2 \pi / 4} \tag{3}$$

where D is the cross-section diameter and ℓ is the coil length. In Equation 3 it is shown that the magnetic reluctance to the flux outside interior of a solenoid is equal to the reluctance of a segment of length $\Delta\ell = 0.45 D_1$ from an indefinitely long coil. On the basis of (2) it is concluded that total magnetic reluctance (R_m) for the solenoid of length (ℓ), equally to solenoid magnetic reluctance of fictitious length $\ell_f = \ell + 0.45 D_1$. Equation 3 shows that the reluctance to the flux outside the interior of the coil has practically a constant value for the coil of the given diameter value (D=const) and the different length value ($\ell \neq$ const).

Based on of the author's results [4], the mutual inductance (M) of coaxial coils with circular cross-section depends on the cross-section area of the inner coil ($A_2 = \pi D_2^2/4$):

$$M = \mu_0 \frac{D^2 \pi}{4(\ell_1 + 0.45 D_1)} \tag{4}$$

where

ℓ_1, D_1 - length and diameter of the outer coil,

ℓ_2, D_2 - length and diameter of the inner coil.

The expression for inductance (L) of the rectangular cross-section cylindrical coil is written in form [5]:

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$$L_{\Pi} = N^2 \mu_0 \frac{bh}{l} \cdot K_{1\Pi} \tag{5}$$

In (3), besides physical and geometrical values (μ_0, N, b, h, ℓ), coefficient K_1 =f(h/l, h/b) appears representing the decreasing inductance due to reluctance to the flux outside the interior of the solenoid. This coefficient sets values in the form of graphs as a function of the given parameters (fig. 1). It aggravates observation of the functional dependence of inductance on the solenoid dimensions. In a similar way it can be proven that the value of magnetic reluctance (ΔR_m) to the flux outside the solenoid interior, of the rectangular cross-section and the given ratio of the value of the shorter and longer side (b/h), is almost invariable with the change in the coil length.

Furthermore, based on the similarity of expressions for the value of reluctance to the flux outside the solenoid interior of the circular and rectangular cross-section, a corresponding expression would be established for calculation of the rectangular cross-section of a solenoid total reluctance (R_m).

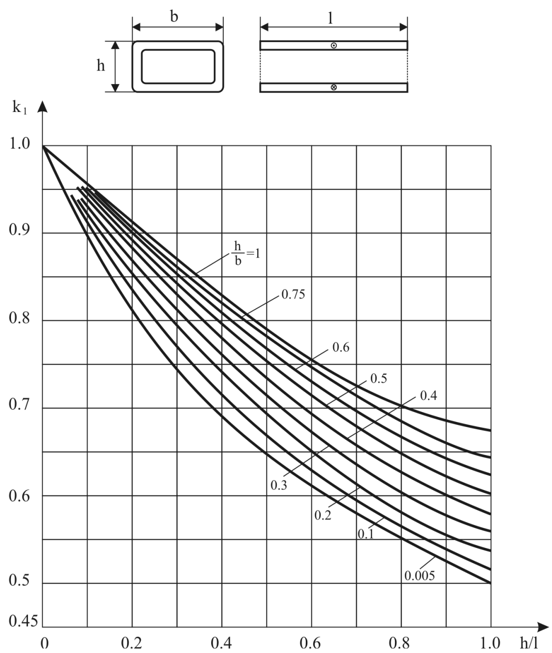


Figure 1. Correction coefficient K_1 =f(h/l, h/b) for calculation of inductance of a rectangular cross-sectional solenoid with ℓ > h or h/ℓ ≤ 1 (long coil), [5].

2 Calculation of Inductance and Reluctance of a Rectangular Cross-section Coil

It is shown that it is practical to define certain reluctances for a rectangular cross-section coil (Fig.2) and to give a corresponding expression for them:

$$R_{ml} = \frac{1}{\mu_0} \cdot \frac{\ell}{bh} \tag{6}$$

$$R_m = \frac{1}{\mu_0} \cdot \frac{\ell}{bh} \cdot \frac{1}{K_1} \tag{7}$$

$$R_m = R_{m0} + \Delta R_{ml} \tag{8}$$

$$\Delta R_m = \frac{1}{\mu_0} \cdot \frac{\ell}{bh} \cdot \frac{1 - K_1}{K_1} \tag{9}$$

$$R_{mh} = \frac{1}{\mu_0} \cdot \frac{h}{bh} \tag{10}$$

Where:

$R_{m\ell}$ - reluctance of a segment of length (ℓ) from the indefinitely long, rectangular cross-section solenoid,

$R_{m\ell}$ - reluctance of a solenoid of finite length (ℓ),

ΔR_m=R_m-$R_{m\ell}$ - reluctance to flux outside the interior of the solenoid, i.e. on the so-called back path of the flux (Fig. 2),

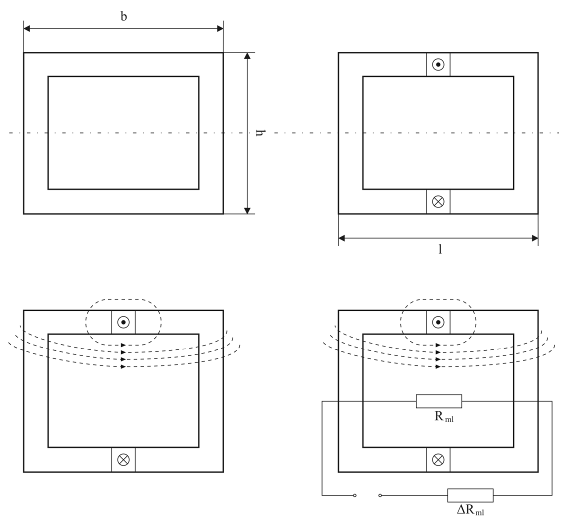
R_{mh}- the same as $R_{m\ell}$, but for a segment of length ℓ=h,

b, h – longer (b) and shorter (h) side of cross-section,

ℓ -length of a given solenoid,

K_1 =f(h/l, h/b) – coefficient of the decreased solenoid inductance of length (ℓ), Fig.1 [5], and

μ_0 – magnetic permeability of air.



(b)

Figure 2. (a) Coil of a rectangular cross-section with (b) realistic of the magnetic field, and (c) corresponding magnetic circuit.

Taking the values for K_1 from the diagram (Fig. 1), the values of R_m, ΔR_m and R_{mh} are calculated. Ratios R_m/R_{mh} and $\Delta R_m/R_{mh}$ are given in table 1. Value R_{mh}, calculated with (10), is taken for a relative unit. As values of the given ratio $\Delta R_m/\Delta R_{mh} \approx$ Const (table A1 in Appendix, column 5) for the definite value ratio of *h/b*, it is concluded that $(\Delta R_m/\Delta R_{mh})_{med}$ =f(h/b).

If is introduced new coefficient K_2 =f(h/b):

$$K_{2\Pi} = (\frac{\Delta R_m}{\Delta R_{mh}})_{med} \tag{11}$$

On the basis values $(\Delta R_m/\Delta R_{mh})_{med}$ =f(h/b) given in Table 1, the corresponding dependence of K_2 =f(h/b) is shown in Fig. 3.

Table 1: Corresponding values K_2 and $K_{2,C}$

h/b	$K_{2\Pi}=(\frac{\Delta R_m}{R_{mh}})_{med}$	$K_{2\Pi,C}=\frac{1}{1+h/b}$	E_{K2C}%
0.100	0.9662	0.9091	-5.90%
0.200	0.8649	0.8333	-3.70%
0.300	0.7884	0.7692	-2.40%
0.400	0.7189	0.7143	-0.60%
0.500	0.6599	0.6667	1.00%
0.600	0.6057	0.6250	3.20%
0.675	0.5821	0.5970	2.50%
0.750	0.5603	0.5714	1.80%
0.875	0.5360	0.5333	0.50%
0.900	0.5312	0.5274	-0.70%
1.000	0.5126	0.5000	-2.50%

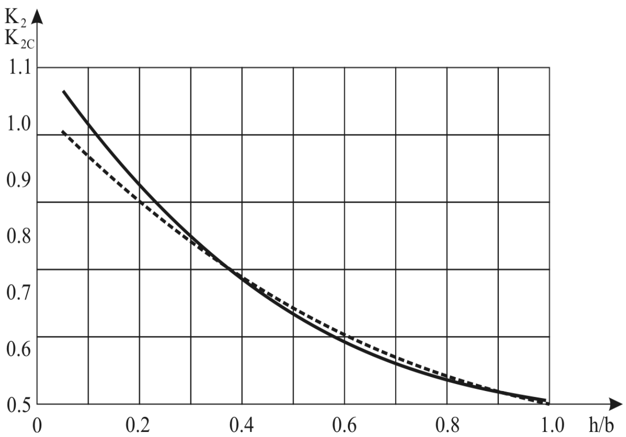


Figure 3. Dependence of inductance coefficients:
— K_2 =f(h/b) for the rectangular cross-section coil, and
---$K_{2,C}$≈b/(b+h) given in the simple relation.

It is shown that the values R_m, ΔR_m and L may be defined with simple relation by introducing coefficient (K_2). Reluctance to the flux outside the interior coil (ΔR_m), i.e. on the so called back path of the flux, is

$$\Delta R_{m\Pi} = \frac{1}{\mu_0} \cdot \frac{h}{bh} \cdot K_{2\Pi} \tag{12}$$

On the basis Equations 11 and 12 we obtain

$$R_{m\Pi} = \frac{1}{\mu_0} \cdot \frac{1}{bh} \cdot (\ell + K_{2\Pi} h) \tag{13}$$

and K_2 are the values given in Table 1 and Fig. 3.

Values $K_{2\Pi} / \sqrt{A_\Pi} = K_{2\Pi} h / \sqrt{bh} = K_{2\Pi} \cdot \sqrt{h/b}$, represented in the increasing magnetic reluctance in a relative unit for the coil of given area ($A_\square$=bh) of a rectangular cross-section, are given in Table 2.

Table 2: Values $K_{2\Pi} h / \sqrt{bh} = K_{2\Pi} \cdot \sqrt{h/b}$

h/b	0.1	0.2	0.3	0.4	0.5
$K_{2\Pi} h / \sqrt{bh}$	0.305	0.387	0.432	0.455	0.467
h/b	0.6	0.675	0.75	0.875	1
$K_{2\Pi} h / \sqrt{bh}$	0.469	0.478	0.485	0.501	0.512

The value for increasing the magnetic reluctance in a relative unit, with a coil of a circular cross-section on the basis of equation (3), is

$$K_{2O} / \sqrt{A_O} = 0.45D / (0.5D\sqrt{3.1459}) = 0.508$$

where $A_\square$=πD²/4 is the area of a circular cross-section.

On the basis of the given value (0.508) and data in Table 2, it is concluded that the coil with a greater ratio of the longitudinal/transversal dimensions of cross-section, i.e. with b/h≥ 2 (or h/b≤ 0.4), should rather be approximated with the rectangular cross-section than with the circular cross-section, as it is usually done.

In this paper is shown that the functional dependence K_2=f(h/b) may be expressed, with a satisfactory correctness, analytically by using the fellow relation:

$$K_{2\Pi} \approx \frac{b}{b+h} \tag{14}$$

Comparing the values in columns 5, 6 and 7 (Table A1 in Appendix), it is clearly seen that expression (14) gives the values for K_2. When expression (14) for K_2 is inserted into the expressions for R_m, ΔR_m and L, the following is obtained

$$\Delta R_{m\Pi} = \frac{1}{\mu_0} \cdot \frac{h}{bh} \cdot \frac{b}{b+h} \quad , \qquad \ell \geq h \geq b \tag{15}$$

$$R_{m\Pi} = \frac{1}{\mu_0} \cdot \frac{1}{bh} \cdot (\ell + h\frac{b}{b+h}) \ , \quad \ell \geq h \geq b \tag{16}$$

$$L_\Pi = N^2 \mu_0 \frac{bh}{l + h \cdot \frac{b}{b+h}} \quad , \qquad \ell \geq h \geq b \tag{17}$$

where (*h)* always denotes the shorter side of the rectangular cross-section and (b) the longer side. As cross section area, A_Π = bh, so

$$L_\Pi = N^2 \mu_0 \frac{A_\Pi}{l} \frac{1}{l + \frac{h}{l} \cdot \frac{b}{b+h}} \quad , \qquad \ell \geq h \geq b \tag{18}$$

Corresponding inductance coefficient $K_{1\Pi}$ in Equations 5 and 18 is:

$$K_{1\Pi} = \frac{1}{l + \frac{h}{l} \cdot \frac{b}{b+h}} , \qquad \ell \geq h \geq b \tag{19}$$

Example 1:

Calculate the value of the inductance of a rectangular cross-section **coil length** (l=30 cm, b=15 cm, h=10 cm), a) by using the procedure given in this paper, Equation 19, and b) the improved - accuracy procedure given in [1, Section 9-3].

a) Calculation inductance by using (19):

$$L_{\Pi} = 20^2 \cdot 4\pi \cdot 10^{-7} \frac{15 \cdot 10 \cdot 10^{-4}}{30 \cdot 10^{-2}} \cdot \frac{1}{1 + \frac{10}{30} \cdot \frac{15}{15+10}}$$

$$L_{\Pi} = 0.20944 \cdot 10^{-4} H$$

b) Calculation made with the improved – accuracy procedure given in [1, Section 9-3]:

Value inductance coefficient $K_{1\Pi}$

$K_{1\Pi} = 1 - \alpha_1(b/c) + \alpha_2(b/c)^2 - \alpha_4(b/c)^4 + \alpha_6(b/c)^6 - \alpha_6(b/c)^8 + ...$

$K_{1\Pi} = 1 - 0.3824 \cdot 0.5 + 0.1031 \cdot 0.5^2 - .0128 \cdot 0.5^4 + 0.0042 \cdot 0.5^6 - 0.0030 \cdot 0.5^8 + ...$

$K_{1\Pi} = 0.83625$

Coefficient values α_1, α_2, α_4, α_6 and α_8 are determined in the data given in Table 9-4 [1].

The corresponding inductance values, calculated with (5), are:

$$L_{\Pi} = 20^2 \cdot 4\pi \cdot 10^{-7} \cdot \frac{15 \cdot 10 \cdot 10^{-4}}{30 \cdot 10^{-2}} \cdot 0.83625$$

$$L_{\Pi} = 0.21017 \cdot 10^{-4} H$$

The difference in the results calculation obtained by using simple Equation (19), i.e. the error, is only 0.35%.

3 Calculation of Inductance of the Short Cylindrical Coils of any Cross-section

For the perimeter of rectangular contour cross-section $P_{\square}=2(b+h)$, expression (15) gets the form:

$$\Delta R_{m\Pi} = \frac{1}{\mu_0} \frac{2}{P_{\Pi}}, \qquad \ell \geq h \geq b \tag{20}$$

meaning that the corresponding magnetic conductance to the flux outside the inner part of coil is

$$\Delta \Lambda_{m\Pi} = \mu_0 \cdot \frac{P_{\Pi}}{2}, \qquad \ell \geq h \geq b \tag{21}$$

On the basis of Equation 3, for the magnetic reluctance to the flux outside the inner part of a circular cross-section coil with perimeter $P_o = D\pi$, the following is obtained

$$\Delta R_{mO} = \frac{1}{\mu_0} \cdot \frac{1.8}{P_o} \tag{22}$$

i.e., the corresponding magnetic conductance is:

$$\Delta \Lambda_{mO} = \mu_0 \cdot \frac{P_o}{1.8} \tag{23}$$

From the expressions (21) and (23) it is seen **that the magnetic conductance to the flux outside the inner part of the coil is directly proportional to the perimeter of the contour of the given cross-section.** Coefficients of proportionality are in both cases slightly different and their values are practically equal for short coils ($\ell \leq h \leq b$). In both cases this value is 1.8. On this basis and using Equation 18, expression is obtained for calculations inductance of short coils ($\ell \leq h \leq b$) with a rectangular cross-section:

$$L_{\Pi} = N^2 \mu_0 \frac{A_{\Pi}}{l} \frac{1}{l + \frac{h}{l} \cdot 0.9 \frac{b}{b+h}}, \quad \ell \leq h \leq b \tag{24}$$

The corresponding inductance coefficient $K_{1\Pi}$ in equations (24) is

$$K_{1\Pi} = \frac{1}{l + \frac{h}{l} \cdot 0.9 \cdot \frac{b}{b+h}}, \qquad \ell \leq h \leq b \tag{25}$$

The accuracy of Equations 24 and 25 is checked by investigating of any short coil ($\ell < h < b$).

Example 2:

Calculate the value inductance of a rectangular cross-section of **coil short** (ℓ =2.5 cm, b=15 cm, h=10 cm), a) by using the procedure given in this paper, Equation (24), and b) the improved - accuracy procedure given in [1, Section 9-3].

a) Calculation inductance with Equation 24

$$L_{\Pi} = 25^2 4\pi \cdot 10^{-7} \frac{15 \cdot 10 \cdot 10^{-4}}{2.5 \cdot 10^{-2}} \cdot \frac{1}{1 + \frac{10}{2.5} \cdot 0.9 \cdot \frac{15}{15+10}}$$

$$L_{\Pi} = 1.4905 \cdot 10^{-4} H$$

b) Calculate inductance by using the improved-accuracy procedure given in [1, Section 9-3]:

$$L_{\Pi} = (\mu_0 / 6.28) \cdot N^2 \cdot \left[b \cdot (\varphi_C - \psi_C) + h \cdot (\varphi_b - \psi_b) \right]$$

$$L_{\Pi} = (4 \cdot 3.14 \cdot 10^{-7} / 6.28) \cdot 25^2 \cdot \left[0.15 \cdot (6.07 - 1.30) + 0.1 \cdot (5.40 - 0.64) \right]$$

$$L_{\Pi} = 1.50 \cdot 10^{-4} H$$

With fig.9-3 and fig.9-4 [1]:

- φ_c =6.07 and ψ_c =1.30, for b/ℓ =6 and h/ℓ =4, and
- φ_B =5.40 and ψ_B =0.64, for b/ℓ =6 and h/ℓ =4.

The difference in the results calculation obtained by using simple Equation (24), i.e. the error, is only 0.637%.

Considering the above and fact and the fact that most irregular cross-section coil may approximate pretty well to the rectangular cross-section, in the sense of approaching the contour in the shape, expressions (22) and (23) may be given a general significance.

By comparing Equations 22 and 24, the conclusion can be drawn that the recommendation from literature to approximate the short coil of the irregular cross-section with the coil of the circular cross-section of the same area, is only partly valid. It is more correct that for the magnetic reluctance of short coils the following is determined:

- partial reluctance ($R_{m\ell}$) of a segment of length (ℓ) from indefinitely long, and
- reluctance ΔR_m to the flux outside the interior of the coil, by using Equation (22).

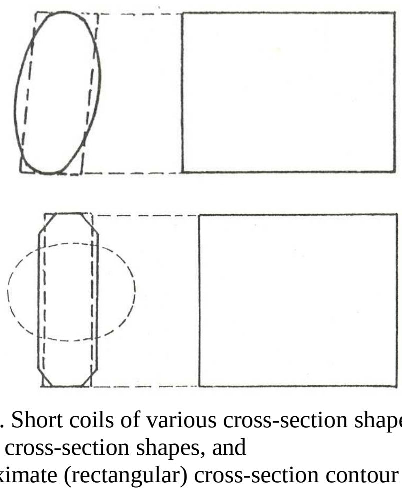
Total reluctance $R_m = \Delta R_m + R_{m0}$ of a short coil of finite length (ℓ) is

$$R_m = \frac{1}{\mu_0} \cdot \frac{\ell}{A} + \frac{1}{\mu_0} \cdot \frac{P}{1.8} \tag{26}$$

The corresponding inductance is

$$L = N^2 \mu_0 \cdot \frac{1}{\ell / S + P / 1.8}, \quad \ell \le \mathrm{h} \le \mathrm{b} \tag{27}$$

It means that approximation of the coil of an irregular cross-section with the coil of a regular cross-section is correct if the two cross-sections have the same values of areas and perimeter (Fig. 4a, b). Approximation with the same area circular cross-section leads to an error in determination of ΔR_m value (Fig. 4a) because of the difference in the perimeter of the circular and rectangular contour. Sometimes, the approximation with the rectangular cross-section may be erroneous (Fig. 4b), although the approximated rectangle and actual cross-sections have the same area values (A), because the actual cross-section has the contour of a longer length. Therefore, the determination of ΔR_m according to the approximation (Fig. 4a) would lead to an error. This confirms that it is more correct to determine R_m and L according to expressions (26) and (27).



(a)

(b)

Figure 4. Short coils of various cross-section shapes:
─ actual cross-section shapes, and
- approximate (rectangular) cross-section contour

5 Conclusion

In this paper simplified analytical expressions to be used in determination of magnetic reluctance and inductance of short cylindrical coils of rectangular-cross section. They are found to be extremely useful when analyzing of the impact of the cross-section shape on the values of magnetic reluctance and inductance of short cylindrical coils.

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Miloje Kostic was born in 1950, in Bioska, Serbia,. He received his B.E. and M.E.E. degrees in electrical engineering from the University Belgrade, Serbia, in 1974 and 1983, respectively, and his Ph.D. degree from University Kragujevac, Serbia, in 1990.

From 1974 to 1991 he worked with the Industrial plant "Prvi partizan", Uzice, Serbia. In 1991, he joined the "Nikola Tesla" Electrical Institute, Belgrade, where he is currently Senior Consultant in the Power System Department.

His fields of interest include Theoretical electrical engineering, Induction heating, Industrial Energy and Electrical Machines. He has written about 115 technical papers, these published in journals and conference proceedings, three books and is the owner of five patents.

Appendix

Table A1: Calculated values of R_m/R_{mh}, $\Delta R_m/R_{mh}$ and $(\Delta R_m/\Delta R_{mh})_{med} = f(h/b)$

h/b	h/ℓ	K1	R_m/R_{mh}	$\Delta R_m/R_{mh}$	$K_{2\square,med}$	$K_{2\square}=b/(b+h)$
0.1	1.0	0.519	1.927	0.927		
0.1	0.9	0.539	2.061	0.950		
0.1	0.8	0.565	2.212	0.962		
0.1	0.7	0.596	2.397	0.968		
0.1	0.6	0.631	2.641	0.975		
0.1	0.5	0.671	2.981	0.981		
0.1	0.4	0.715	3.497	0.997		
0.1	0.3	0.776	4.296	0.962		
0.1	0.2	0.837	5.974	0.974		
0.1				8.696	0.966	0.909
0.2	1.0	0.545	1.835	0.835		
0.2	0.9	0.558	1.991	0.880		
0.2	0.8	0.580	2.155	0.905		
0.2	0.7	0.618	2.312	0.883		
0.2	0.6	0.656	2.541	0.874		
0.2	0.5	0.695	2.878	0.878		
0.2	0.4	0.745	3.356	0.856		
0.2	0.3	0.795	4.193	0.860		
0.2	0.2	0.860	5.814	0.814		
0.2				7.784	0.865	0.833
0.3	1.0	0.564	1.773	0.773		
0.3	0.9	0.581	1.912	0.801		
0.3	0.8	0.612	2.042	0.792		
0.3	0.7	0.636	2.246	0.818		
0.3	0.6	0.672	2.480	0.813		
0.3	0.5	0.717	2.789	0.789		
0.3	0.4	0.766	3.264	0.764		
0.3	0.3	0.817	4.080	0.747		
0.3	0.2	0.868	5.760	0.760		
0.3				7.058	0.784	0.769
0.4	1.0	0.579	1.727	0.727		
0.4	0.9	0.602	1.846	0.735		
0.4	0.8	0.626	1.997	0.747		
0.4	0.7	0.659	2.168	0.739		
0.4	0.6	0.692	2.408	0.742		
0.4	0.5	0.734	2.725	0.725		
0.4	0.4	0.779	3.209	0.709		
0.4	0.3	0.831	4.011	0.678		
0.4	0.2	0.882	5.669	0.669		
0.4				6.470	0.719	0.714
0.5	1.0	0.603	1.658	0.658		
0.5	0.9	0.618	1.798	0.687		
0.5	0.8	0.647	1.932	0.682		
0.5	0.7	0.676	2.113	0.685		
0.5	0.6	0.713	2.338	0.671		
0.5	0.5	0.753	2.656	0.656		
0.5	0.4	0.794	3.149	0.649		
0.5	0.3	0.839	3.973	0.640		
0.5	0.2	0.891	5.612	0.612		
0.5				5.939	0.660	0.667
0.6	1.0	0.623	1.605	0.605		
0.6	0.9	0.641	1.733	0.622		
0.6	0.8	0.664	1.883	0.633		
0.6	0.7	0.696	2.053	0.624		
0.6	0.6	0.727	2.293	0.626		
0.6	0.5	0.765	2.614	0.614		
0.6	0.4	0.808	3.094	0.594		
0.6	0.3	0.851	3.917	0.584		
0.6	0.2	0.901	5.549	0.549		
0.6				5.451	0.606	0.625
0.675	1.0	0.632	1.582	0.582		
0.675	0.9	0.650	1.709	0.598		
0.675	0.8	0.673	1.857	0.607		
0.675	0.7	0.705	2.026	0.598		
0.675	0.6	0.737	2.261	0.595		
0.675	0.5	0.772	2.591	0.591		
0.675	0.4	0.814	3.071	0.571		
0.675	0.3	0.855	3.899	0.565		
0.675	0.2	0.904	5.531	0.531		
0.675				5.239	0.582	0.597
0.75	1.0	0.641	1.560	0.560		
0.75	0.9	0.659	1.686	0.575		
0.75	0.8	0.681	1.836	0.586		
0.75	0.7	0.713	2.004	0.575		
0.75	0.6	0.746	2.234	0.567		
0.75	0.5	0.779	2.567	0.567		
0.75	0.4	0.819	3.053	0.553		
0.75	0.3	0.859	3.880	0.547		
0.75	0.2	0.907	5.513	0.513		
0.75				5.043	0.560	0.571
0.875	1.0	0.658	1.520	0.520		
0.875	0.9	0.671	1.656	0.545		
0.875	0.8	0.692	1.806	0.556		
0.875	0.7	0.720	1.984	0.556		
0.875	0.6	0.752	2.216	0.550		
0.875	0.5	0.784	2.551	0.551		
0.875	0.4	0.824	3.034	0.534		
0.875	0.3	0.864	3.858	0.525		
0.875	0.2	0.911	5.488	0.488		
0.875				4.824	0.536	0.533
1	1.0	0.675	1.481	0.481		
1	0.9	0.682	1.629	0.518		
1	0.8	0.703	1.778	0.528		
1	0.7	0.727	1.965	0.536		
1	0.6	0.758	2.199	0.532		
1	0.5	0.789	2.535	0.535		
1	0.4	0.829	3.016	0.516		
1	0.3	0.869	3.836	0.502		
1	0.2	0.915	5.464	0.464		
1				4.614	0.513	0.500