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Mathematical Model and Steady-State Operational Characteristics of a Unified Power Flow Controller

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Abstract. The paper presents a mathematical model and steady-state operational characteristics of a Unified Power Flow Controller (UPFC). The mathematical description is based on the transformation of a three-phase system into an orthogonal synchronously rotating coordinate system, in which, under steady-state conditions, all the system quantities are constant values. Thus, at given parameters, it is generally possible to determine the dimensions of the device. The analysis of the steady-state operational characteristics resulted in key findings enabling derivation of control algorithms and further examination of the device operating under dynamic conditions.

Key words: FACTS, UPFC, mathematical model, steady-state operational characteristics

Matematični model in stacionarne obratovalne karakteristike univerzalne naprave za spreminjanje pretokov moči

Povzetek. Članek predstavlja matematični model in stacionarne obratovalne karakteristike univerzalne naprave za spreminjanje pretokov moči v elektroenergetskih omrežjih (Unified Power Flow Controller – UPFC). Matematični opis temelji na pretvorbi trifaznega sistema v ortogonalni rotirajoči koordinatni sistem, v katerem v stacionarnih razmerah vse sistemske veličine zavzamejo stalne vrednosti. Tako je na splošno mogoče pri danih parametrih določiti dimenzije naprave. Z analizo stacionarnih obratovalnih karakteristik pridemo do ključnih ugotovitev za izpeljavo regulacijskih algoritmov in za nadaljnjo obravnavo naprave v dinamičnih razmerah obratovanja.

Ključne besede: FACTS, UPFC, matematični model, stacionarne obratovalne karakteristike

1 Introduction

In recent years, the development of the semiconductor technology has led to the use of power electronics in electric power devices, particularly in Flexible AC Transmission Systems (FACTS) apparatuses [1-3]. Advantages of the "FACTS" devices are primarily rapid response and enhanced flexibility. Flexible electric power systems allow for improved transmission capability when the distances between the production and consumption centers are long, and better power flow control along parallel paths in relatively meshed networks. A significant aspect of devices based on power electronics is the fact that they are more expensive than similar conventional equipment, but they allow numerous functions that cannot be performed with conventional devices. As a result of their nature, power electronics equipment can be applied when one or more of the following requirements have to be met: rapid response, frequent output variations, smoothly adjustable output.

High voltage DC transmissions (HVDC), static var compensators (SVC) with thyristor-controlled reactive elements, thyristor-controlled series compensators (TCSC) and classical battery energy storage systems have been used in practice for quite some time. A special group includes compensators based on power voltage source converters (VSC), i.e. Static Compensator (StatCom), Static Synchronous Series Compensator (SSSC) and Unified Power Flow Controller (UPFC) that are operating experimentally. The most versatile FACTS device is the UPFC [4]. It enables rapid and precise control of active and reactive power flows in a transmission line.

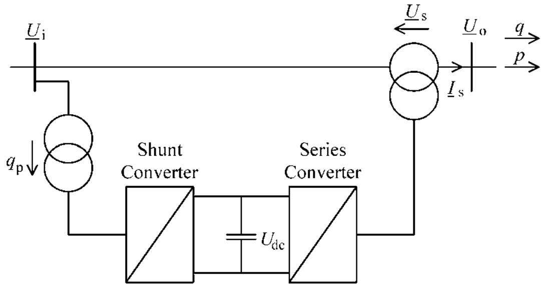

Figure 1. Basic scheme of a UPFC

The UPFC consists of two voltage source converters using gate turn-off thyristors (GTO). They operate from

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a common dc-circuit consisting of a dc-storage capacitor (Fig. 1). The UPFC can be described as consisting of a shunt and a series branch. Each converter independently generates or absorbs the reactive power. This arrangement enables a free flow of the active power in either direction between the ac terminals of the two converters. The function of the shunt converter is to supply or absorb the active power demanded by the series branch. It is connected to the ac terminal through a shunt-connected transformer. If required, it may also inject the leading or lagging reactive power directly into the connection busbar. The second series connected converter provides the main function of the UPFC by injecting an ac voltage with a controllable magnitude and phase angle. The transmission line current flows through this voltage source resulting in an active and reactive power exchange with the ac system.

Our analysis of the UPFC is based on the transformation of alternating three-phase quantities into the rotating reference frame where the quantities become constant values [5]. If in the state space equations in the said coordinate system the dynamic part is neglected or assumed to equal zero, an algebraic equation is obtained from the differential one. This equation enables the calculation of different steady-state operational points of the device depending on a certain quantity [6].

All the steady-state operational characteristics of the UPFC can be divided into two groups. The first group consists of the system quantity characteristics, i.e. currents depending on the adjustable parameters of a voltage source converter (amplitude and phase angle). In this way we can observe the effect of the device operation on the system conditions. The second group consists of the characteristics of adjustable parameters of a voltage source converter depending on the system quantities. Thus the required values of the adjustable parameters of the device needed for achieving a certain current condition in the system can be studied.

2 Mathematical Model of UPFC

The equivalent circuit of the UPFC is shown in Fig. 2. On the ac side, the device can be represented by sinusoidal voltage sources (switching harmonics are ignored) with a controllable amplitude and phase angle. The circuit also consists of series and shunt impedances representing coupling transformers. The influence of both converters connected in series and shunt to the dc system can be represented by two current sources in the common dc circuit connected to the condenser C. The shunt connection of resistance R_C enables representation of losses in the dc circuit.

Our derivations and mathematical descriptions are based on the transformation of a balanced three-phase system (L1, L2, L3) into an orthogonal synchronously rotating (ω_0=2π·50) coordinate system (d,q). The d-axis of this new coordinate system coincides with the input voltage vector $\underline{u}_i$. For the purposes of further derivation of our mathematical model, the per-unit system was adopted (Eq. (1)) where i_B and u_B are the base values and ω_B is the synchronous angular speed of the fundamental network voltage component [7]. A 50 Hz base frequency is assumed.

$$
\begin{gathered}
i'_{\mathrm{px}} = \frac{i_{\mathrm{px}}}{i_{\mathrm{B}}};\ i'_{\mathrm{sx}} = \frac{i_{\mathrm{sx}}}{i_{\mathrm{B}}};\ i'_{\mathrm{dc}} = \frac{i_{\mathrm{dc}}}{i_{\mathrm{B}}} \\
u'_{\mathrm{px}} = \frac{u_{\mathrm{px}}}{u_{\mathrm{B}}};\ u'_{\mathrm{sx}} = \frac{u_{\mathrm{sx}}}{u_{\mathrm{B}}};\ u'_{\mathrm{dc}} = \frac{u_{\mathrm{dc}}}{u_{\mathrm{B}}} \\
u'_{\mathrm{ix}} = \frac{u_{\mathrm{ix}}}{u_{\mathrm{B}}};\ u'_{\mathrm{ox}} = \frac{u_{\mathrm{ox}}}{u_{\mathrm{B}}} \\
z_{\mathrm{B}} = \frac{u_{\mathrm{B}}}{i_{\mathrm{B}}};\ L'_{\mathrm{p}} = \frac{\omega_{\mathrm{B}} L_{\mathrm{p}}}{z_{\mathrm{B}}};\ R'_{\mathrm{p}} = \frac{R_{\mathrm{p}}}{z_{\mathrm{B}}} \qquad (1) \\
L'_{\mathrm{s}} = \frac{\omega_{\mathrm{B}} L_{\mathrm{s}}}{z_{\mathrm{B}}};\ R'_{\mathrm{s}} = \frac{R_{\mathrm{s}}}{z_{\mathrm{B}}} \\
C' = \frac{1}{\omega_{\mathrm{B}} C\, z_{\mathrm{B}}};\ R'_{\mathrm{c}} = \frac{R_{\mathrm{c}}}{z_{\mathrm{B}}};\ \omega_{\mathrm{B}} = 2\pi \cdot 50 \\
\mathrm{x} = \mathrm{d,q}
\end{gathered}
$$

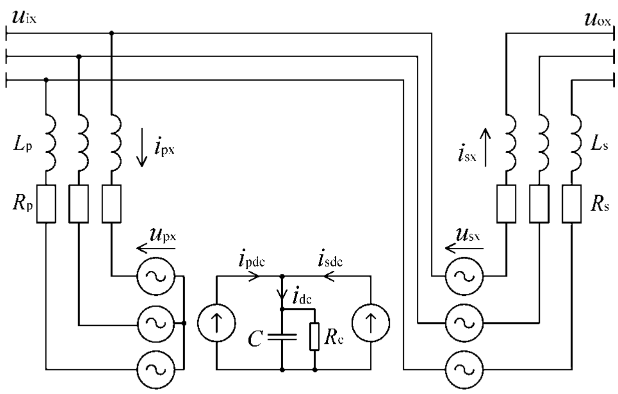

Figure 2. Equivalent circuit of the UPFC using sinusoidal voltage sources

Components of the both converter output voltages depend on the dc voltage. We can write Eqs. (2) and (3) where k_p and k_s are factors relating the dc and ac side voltages of the series and shunt converters, respectively. These factors are functions of the transformer ratios as well as of the particular converter topology being employed. Angles δ_p and δ_s represent the phase shift of each converter output voltage from the reference position. The control factors m_p (shunt converter) and m_s (series converter) can take any value between zero and one as the converter output voltage is varied over its controllable range.

$$u'_{pd} = u'_{dc} k_p m_p \cos\delta_p = u'_{dc} k_p d_{pd}$$
$$u'_{pq} = u'_{dc} k_p m_p \sin\delta_p = u'_{dc} k_p d_{pq} \quad (2)$$

$$u'_{sd} = u'_{dc} k_s m_s \cos\delta_s = u'_{dc} k_s d_{sd}$$
$$u'_{sq} = u'_{dc} k_s m_s \sin\delta_s = u'_{dc} k_s d_{sq} \quad (3)$$

Adjustable parameters of the converter output voltages m_p, δ_p, m_s and δ_s can be transformed into the average switching functions d_{pd} in the direction of the d-axis and d_{pq} in the direction of the q-axis for the shunt converter, and d_{sd}, d_{sq} for the series converter. The resulting common balance equation for the active power in terms of d-q components of the quantities is shown in Eq. (4).

$$u'_{dc} i'_{dc} = \frac{3}{2}\left(u'_{pd} i'_{pd} + u'_{pq} i'_{pq} + u'_{sd} i'_{sd} + u'_{sq} i'_{sq}\right) \quad (4)$$

In a rotating coordinate system the influence of both converters on the dc-condenser can be represented by a common dc current source (Eq. (5)).

$$i'_{dc} = i'_{pdc} + i'_{sdc} =$$
$$= \frac{3}{2}\left(k_p d_{pd} i'_{pd} + k_p d_{pq} i'_{pq} + k_s d_{sd} i'_{sd} + k_s d_{sq} i'_{sq}\right) \quad (5)$$

The equations for the entire circuit are derived and converted into a state-space formulation in a rotating coordinate system as given in Eq. (6) where p.=d/dt.

$$p.\begin{bmatrix} i'_{pd} \\ i'_{pq} \\ i'_{sd} \\ i'_{sq} \\ u'_{dc} \end{bmatrix} = \mathbf{A}\begin{bmatrix} i'_{pd} \\ i'_{pq} \\ i'_{sd} \\ i'_{sq} \\ u'_{dc} \end{bmatrix} + \begin{bmatrix} \frac{\omega_B}{L'_p} u'_{id} \\ \frac{\omega_B}{L'_p} u'_{iq} \\ \frac{\omega_B}{L'_s}\left(u'_{id} - u'_{od}\right) \\ \frac{\omega_B}{L'_s}\left(u'_{iq} - u'_{oq}\right) \\ 0 \end{bmatrix}$$

where,

$$\mathbf{A} = \begin{bmatrix} -\frac{R'_p \omega_B}{L'_p} & \omega_0 & 0 & 0 & -a_{p1} d_{pd} \\ -\omega_0 & -\frac{R'_p \omega_B}{L'_p} & 0 & 0 & -a_{p1} d_{pq} \\ 0 & 0 & -\frac{R'_s \omega_B}{L'_s} & \omega_0 & -a_{s1} d_{sd} \\ 0 & 0 & -\omega_0 & -\frac{R'_s \omega_B}{L'_s} & -a_{s1} d_{sq} \\ a_{p2} d_{pd} & a_{p2} d_{pq} & a_{s2} d_{sd} & a_{s2} d_{sq} & -\frac{C' \omega_B}{R'_c} \end{bmatrix}$$

where,

$$a_{p1} = \frac{k_p \omega_B}{L'_p},\ a_{p2} = \frac{3 k_p \omega_B C'}{2},\ a_{s1} = \frac{k_s \omega_B}{L'_s},\ a_{s2} = \frac{3 k_s \omega_B C'}{2} \quad (6)$$

3 Steady-State Operational Characteristics

In the calculation of the steady-state operational characteristics of the UPFC, characteristic pu-values for shunt and series system parameters and for the dc-circuit are used (Eq. (7)).

$$R'_p = 0.015\,,\quad L'_p = 0.15\,,\quad R'_s = 0.01$$
$$L'_s = 0.10\,,\quad R'_C = 50\,,\quad C' = 0.5 \quad (7)$$

The amplitude of the shunt connected converter output voltage is given by the product $u'_{dc} k_p m_p$. As already mentioned, the quantity k_p is the factor including the transformer ratio and relating the dc and ac voltage of the observed converter type. This factor can be further divided into component k_{pt}, including the shunt coupling transformer ratio, and into component k_{pp} depending on the converter type. The amplitude is given now by $u'_{dc} k_{pp} k_{pt} m_p$. For the expression $k_{pt} m_p$ a new parameter A_p' is introduced since it is more evident and directly represents the pu-value of the converter output voltage. It is assumed that the product $u'_{dc} k_{pp}$ is chosen so that its value always equals 1. For the voltage source converter in the H-connection the factor k_{pp} equals 4/π. This means that in this case the rated pu-value of the dc voltage equals π/4. According to the above mentioned considerations, δ_p and A_p' appear as two independent adjustable parameters in all calculations of the steady-state operational characteristics for the shunt connected voltage source converter. The latter parameter can also take values exceeding 1. The highest value of the control factor m_p equals 1, which means the factor k_{pt} has to be sufficiently bigger than 1.

The UPFC is located at the sending end of the transmission line. The coordinate system with its d-axis coincides with the position of the voltage vector at the sending end of the line, the voltage vector at the receiving end of the line equals 1 and is shifted by -20° (Eq. (8)). The initial components of the series current (UPFC injected voltage = 0) are i_{sd0}'=3.44 pu and i_{sq0}'=-0.26 pu. The current component in the direction of the d-axis represents the active current flowing through the line and the current component in the q-axis direction represents the reactive component of the said current. Both current components are proportional to the active and reactive power at the sending end of the line. This relation is not valid at the receiving end of the transmission line.

$$u'_{id} = 1\,,\quad u'_{iq} = 0,\ u'_{od} = 0.94\,,\quad u'_{oq} = -0.34$$
$$\delta_i = 0°,\quad \delta_o = -20° \quad (8)$$

For the UPFC, first the characteristics for the operation mode with the constant dc voltage are given, followed by the characteristics for the operation mode with a varying dc voltage.

3.1 Operation with a constant dc voltage

The steady-state characteristics of the UPFC in the operation mode with a constant dc voltage are based on Eq. (6). In Fig. 3, traces of the typical UPFC quantities depending on the phase angle of the series converter output voltage are given. The amplitude of the output voltage of the series branch assumes different constant values. The two series current components have a sinusoidal shape displaced from the zero-crossing position for the initial values of both components. As mentioned earlier, the exchange of active power between the series branch and the network primarily depends on the reactive current component flowing through the transmission line. For the series branch, active power exchange is the responsibility of the shunt converter with its active current component, meaning that the shape of this current component is similar to the series reactive current component. Since the amplitude of the output voltage of the shunt converter is set to 1 pu, the reactive current component also varies if we change the active component of this current.

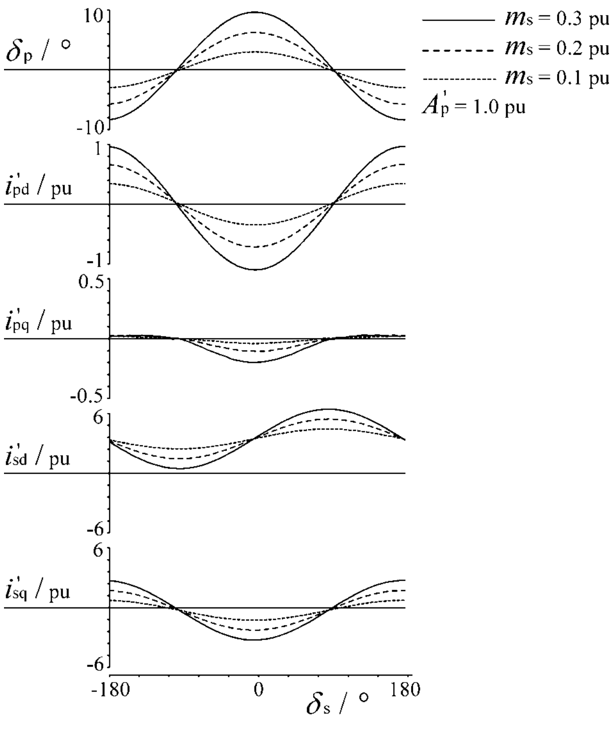

Figure 3. Steady-state operational characteristics of the UPFC when operating with a constant dc voltage depending on phase angle of the series converter output voltage δ_s and for different values of m_s

In Fig. 4, traces of the typical UPFC quantities depending on the phase angle of the series converter output voltage are given. The amplitude of the output voltage of the shunt converter assumes different constant values. As shown, the shunt branch with its active current component supplies or absorbs the active power dictated by the series branch. By varying the amplitude of the shunt converter voltage, the reactive current of the shunt branch varies, too.

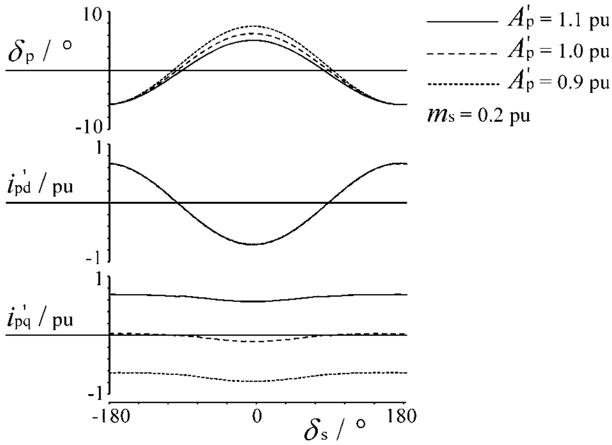

Figure 4. Steady-state operational characteristics of the UPFC when operating with a constant dc voltage depending on phase angle of the series converter output voltage δ_s and for different values of A_p'

In Fig. 5, traces of the typical UPFC quantities depending on the series active current component are given. The series reactive current component assumes different constant values. The strong relation between the series reactive current component and the shunt active current component or the phase shift of the shunt converter output voltage is again evident. The amplitude and the phase shift of the series injected voltage are not suitable control parameters for the series branch of the device.

In Fig. 6, traces of the typical UPFC quantities depending on the series active current component are given, but now the shunt reactive current component assumes different constant values. As shown, the shunt reactive current component can be varied independently with the shunt converter. The shunt active current component exchanges the active power for the series branch in addition to covering losses for the whole device. At an increased absolute value of the shunt reactive current component, the losses of the shunt branch also increase, as demonstrated by the trace of the active current component of the said branch.

3.2 Operation with a variable dc voltage

The UPFC can also operate with a varying dc voltage. In this case, the converter of the shunt branch has only one free parameter. This means that it has a simpler structure, but its operation is more complex from the control point of view. The basic principle is that the series branch has to assume a sufficiently high dc voltage to achieve the biggest amplitude of the output series injected voltage. The dc voltage is lower than the rated value only when the shunt converter operates in the inductive range. The operation of the shunt branch in the capacitive range is not

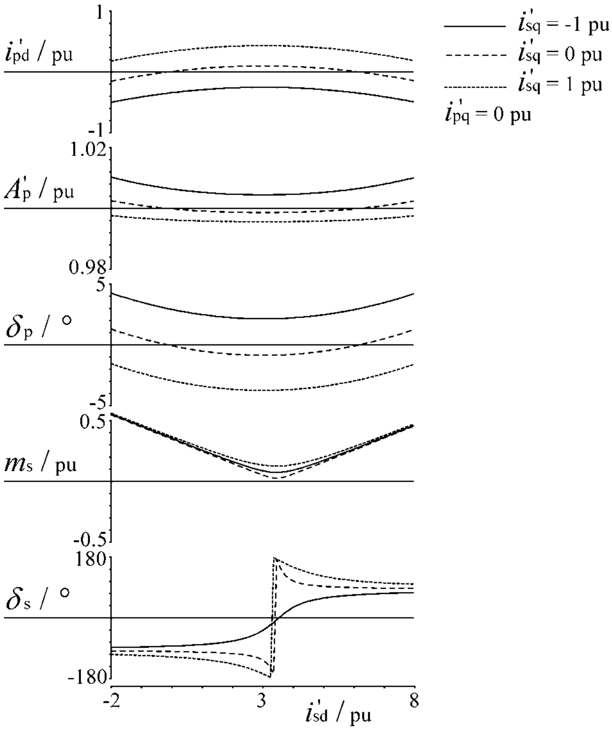

Figure 5. Steady-state operational characteristics of the UPFC when operating with a constant dc voltage depending on series active current component i_{sd}' and for different values of i_{sq}'

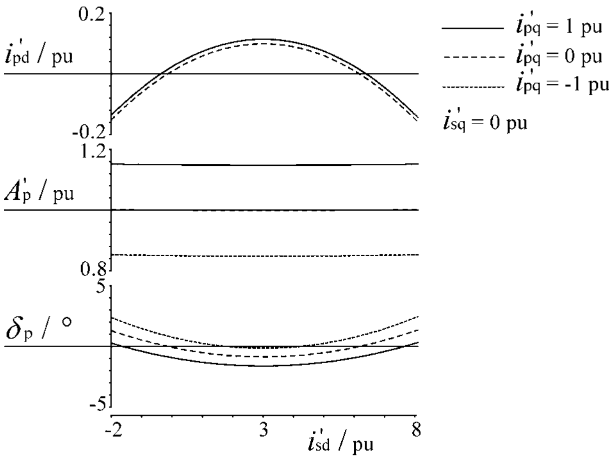

Figure 6. Steady-state operational characteristics of the UPFC when operating with a constant dc voltage depending on series active current component i_{sd}' and for different values of i_{pq}'

problematic, because the dc voltage in this case exceeds the rated value. The steady-state operational characteristics of the UPFC in the operating mode with a varying dc voltage are based on the same equation as in the previous case. In Fig. 7, traces of the typical UPFC quantities depending on the average switching function in the direction of the q-axis are given, the phase shift of the shunt output voltage assumes different constant values and the average switching function in the direction of the d-axis equals zero. By changing the phase shift of the shunt output voltage the dc voltage also changes, which consequently affects the magnitude of the reactive current component of the shunt branch.

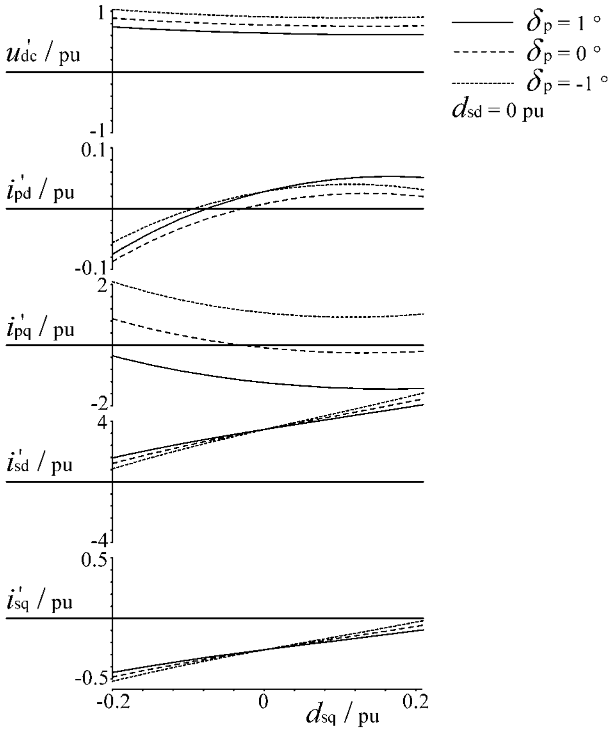

Figure 7. Steady-state operational characteristics of the UPFC when operating with varying dc voltage depending on average switching function d_{sq} for different values of δ_p

In Fig. 8, traces of the typical UPFC quantities depending on the reactive current component of the shunt branch are given and the active current component of the current through the line assumes different constant values. The reactive current component through the line equals zero. This means that the active power exchange between the branches is relatively small. It is evident that by changing the dc voltage, the control factor m_s for the series branch has to be corrected, too. However, changing the active current component of the series branch does not affect the dc voltage, which is primarily dependent on the reactive current component of the shunt branch.

4 Conclusions

The steady-state operational characteristics of the UPFC can be determined using the derived mathematical model or state space equation of the device. The mathematical description is based on the transformation of a three-phase system into an orthogonal synchronously rotating coordinate system, in which, under steady-state conditions, all system quantities are constant values. Thus, at given parameters, it is generally possible to determine the dimensions of the device.

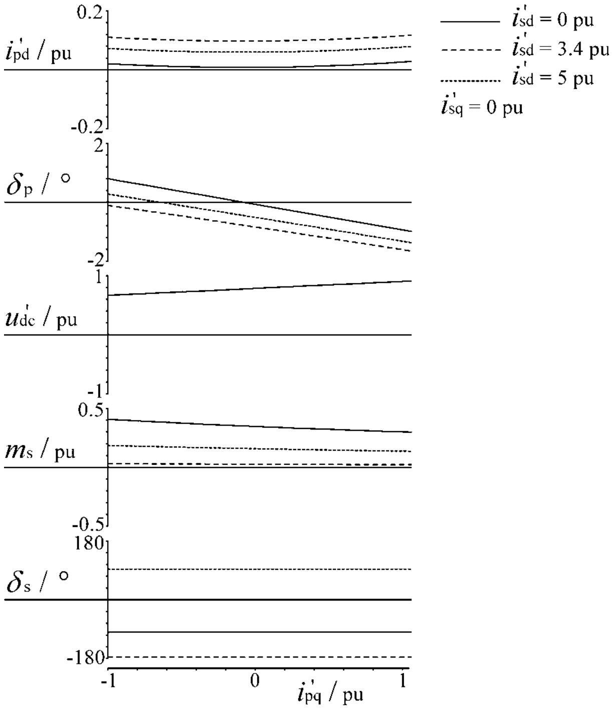

Figure 8. Steady-state operational characteristics of the UPFC when operating with a varying dc voltage depending on shunt reactive current i_{pq}' for different values of i_{sd}'

The analysis of the steady-state characteristics of the shunt connected converter of the UPFC allows us to state that there is an almost linear dependence between the output voltage amplitude of the shunt voltage source converter and the reactive current component, and between the phase angle and the active current component flowing between the shunt converter and the network. In the case of a series connected voltage source converter, this relation is not clearly evident. It is better if we observe the two average switching functions. For the given transmission angle, it is evident from the presented steady-state characteristics that the active component of the series current primarily and almost linearly depends on the average switching function in the direction of the q-axis and that the reactive component of the series current depends on the average switching function in the direction of the d-axis. In a typical transmission system with an exchange of the reactive power between the series converter and the network we can control the flow of the active power through the transmission line. This task can be adequately solved by the series branch alone without the exchange of the active power or by the SSSC.

Since the variation of the reactive current component of the transmission line is related to the exchange of the active power between the series branch and the network, and since the shunt branch compensates the active power inside the device, a very clear relation exists between the reactive series current component and the active current component of the shunt branch. The assessment of the operation mode with a constant dc voltage proves that the reactive shunt current component and the active series current component are primarily independent variables since they can be affected by the reactive power of both converters that is generated or absorbed inside the semiconductor bridge. Finally, we can conclude that the analysis of the steady-state operational characteristics resulted in key findings enabling a further derivation of control algorithms and examination of the UPFC under dynamic operating conditions.

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