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Mathematical Simulation of Stationary Modes of the Induction Motor Under a Cyclically Variable Load

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Abstract. The paper presents algorithms that are based on a mathematical model of the induction motor and developed on the basis of the circuit theory and a projection method of solving the boundary value problem to obtain periodic dependencies of coordinates in a timeless domain and to calculate static characteristics. The steady-state periodic mode is described by a system of differential equations in the *x* and *y* coordinate axes. The magnetization characteristics are used to calculate the electromagnetic parameters of the motor corresponding sections of the magnetic circuit. The rotor winding is supplied in a form of a multi-layered structure formed by dividing the rotor bars by their height into several elements to take into account the current displacement.

Keywords: induction motor, cyclically variable load, steady dynamic mode, boundary value problem, mechanical resonance

Matematična simulacija stacionarnih načinov indukcijskega motorja pri ciklično spremenljivi obremenitvi

V prispevku predstavljamo algoritme, ki temeljijo na matematičnem modelu indukcijskega motorja. Model smo razvili na podlagi teorije vezja in projekcijski metodi reševanja robnega problema, ki omogoča pridobivanje periodičnih odvisnosti koordinat v brezčasni domeni in izračun statičnih lastnosti. Stacionarni periodični način smo opisali s sistemom diferencialnih enačb v koordinatnih oseh x, y. Karakteristike magnetizacije se uporabljajo za izračun elektromagnetnih parametrov motorja, ki ustrezajo odsekom magnetnega vezja. Navitje rotorja je izvedeno v obliki večplastne strukture, ki je oblikovana z razdelitvijo palic rotorja po višini na več elementov, kar omogoča upoštevanje tokovnega premika.

1 INTRODUCTION

Induction electric drives occupy a leading place among a wide class of different electric drives. Their leading position is due to the simplicity of their design and reliability in the operation of induction motors (IM). In practice, technological processes are often characterized by periodic variations in the load torque value as a function of time or the rotor rotation angle. The industry does not produce only general purpose IMs to work in long-term operational modes but also special ones for operation in repeated short-term modes [1 – 3] where each cycle consists of a work time under a constant load and stop time. During operation, the motor's steady temperature is not reached, and the downtime is too short to cool down to the ambient temperature. Also, there are the so-called [4] intermittent modes with cycle times in which the duration of the operating time and pause is too short to achieve thermal equilibrium.

IM designed for operation in a long-term nominal mode can be used for electric drives with a periodically variable load while reducing the nominal power accordingly and checking the overload capacity according to the electromagnetic torque and thermal mode. For this purpose, need to have time dependencies for the currents and flux linkages during the period of a given loading cyclogram to determine the periodic dependencies of the electromagnetic torque, active and reactive powers, and other parameters. These dependencies can only be obtained on the basis of a mathematical model of the drive system with a high level of adequacy, which is adapted to the operating conditions. Mathematical modeling makes it possible to choose an adequate IM and also to develop a control system that ensures the reliable operation of the motor under the specified technological conditions and the highest possible operating efficiency of the electric drive as a whole.

2 STATE OF THE PROBLEM

By analyzing IM processes using classical equivalent schemes [1-4] it is possible to solve only certain problems, such as designing IM with a high probability to operate in a steady state at a constant load. At a periodically variable load, the stationary mode is dynamic. Therefore, to calculate the dynamic modes,

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classical equivalent schemes are not suitable despite numerous proposal methods to improve the definition of the parameters of equivalent schemes in order to adapt them for each specific case.

In the case of a periodic law of the change of the load torque within a period, it can have an arbitrary character, including the pulse (repeated intermittently), which is characterized by a rapid change in the load parameters. The full change cycle (period T_M) consists of two parts: the duration of the load pulse and the pause. The law of the change of the load torque is determined by the technological process, and therefore its period is known. The task of the calculation is to determine the laws of the change of coordinates during the period.

Since under the condition of a periodic torque on the IM shaft, the processes are dynamic, the equations of the electromechanical balance of the IM contours cannot be reduced to the algebraic ones by transforming the coordinates. In any system of coordinate axes, these equations will be differential. Therefore, the task of analyzing the processes in an induction electric drive with periodic disturbances is to solve the DE system, which is nonlinear due to the saturation of the magnetic circuit and current displacement in the rotor bars. The coordinates of the stationary operational mode within the period will change according to the relevant laws. The task of obtaining these dependencies is the goal of the calculation.

The purpose of the work is to develop a mathematical model and algorithm for numerical analysis of the IM dynamic modes of the condition at a cyclically variable load.

3 MATHEMATICAL MODEL

An important issue of mathematical modeling dynamic IM modes is the choice of a suitable system of the coordinate axes to solve assure the model accuracy and adequate results at a minimal cost of the machine time. The IM mathematical model should take into account all the main factors affecting the calculation results, such as the magnetic circuit saturation and the current displacement in the rotor bars. To increase of the calculation accuracy requires a complex IM mathematical model and a fast program. As the number of calculations and the speed of the mathematical modeling program are not reciprocal, a compromise between them needs to be found.

In general, the IM processes can be analyzed with equations written in both physical and transformed coordinate axis. Their most complete representation can be made in physical coordinates, which are widely used to calculate transient processes [5, 6]. At the same time, a transformation of coordinates in the majority of problems important for practice significantly simplifies DE which describes the dynamic IM mode, without reducing the result accuracy while increasing the calculation algorithm efficiency in terms of the number of calculations and the required computer memory. In particular, phase fluxes in orthogonal coordinates are functions of currents only and do not depend on the rotor of rotation angle [7].

Calculation algorithms are based on an IM mathematical model in the *x* and *y* axes at a short-circuited rotor winding developed on the bases of the theory of image vectors [7] and making it possible to consider processes based on the circuit theory. To analyze IM electromagnetic processes, using a mathematical model makes it possible to research processes taking into account the saturation and current displacement in the rotor winding bars with the smallest possible number of calculations. To take into account the saturation, the characteristics of magnetization at a main magnetic flux and scattering fluxes are used. To take into account current displacement, the rotor bars are divided by their height into *k* layers $(2 \le k \le 5)$ this gives *k* windings on the rotor. They are covered by different magnetic scattering fluxes but have a common working magnetic flux. DE of the IM electrical equilibrium powered by a three-phase network with a symmetrical voltage system and operating in a variable load mode has the below form [5 – 7]

$$
\begin{aligned}
&\frac{d\psi_{sx}}{dt} = -\omega_0 \psi_{sy} - r_s i_{sx} + U_m ;\\
&\frac{d\psi_{sy}}{dt} = -\omega_0 \psi_{sx} - r_s i_{sy} ;\\
&\frac{d\psi_{1x}}{dt} = s\omega_0 \psi_{1y} - r_1 i_{1x} ;\\
&\frac{d\psi_{1y}}{dt} = -s\omega_0 \psi_{1x} - r_1 i_{1y}\\
&\qquad\vdots\\
&\frac{d\psi_{kx}}{dt} = s\omega_0 \psi_{ky} - r_k i_{kx} ;\\
&\frac{d\psi_{ky}}{dt} = -s\omega_0 \psi_{kx} - r_k i_{ky} ;
\end{aligned}
\tag{1}
$$

where *sx* and *sy* denote the belongingness of flow couplings (ψ), currents (*i*), and active resistances (*r*) to the corresponding stator circuits; $1x,\ldots,kx$, $1y,\ldots,ky$ – rotor; U_m ω_0 is the amplitude value and angular frequency of the phase voltage of the supply voltage of the stator winding; $\omega = \omega_0(1-s)$ is the angular speed of the rotor rotation expressed in electric radians per second; *s* is the slip.

The DE system (1) is supplemented to the full system by the equation for flux linkages of circuits. They are determined based on the use of magnetization curves by main magnetic flux ψ_μ and winding dissipation currents $\psi_{\sigma s}$ stator and $\psi_{\sigma r}$ rotor

$$\psi_\mu=\psi_\mu(i_\mu),\quad \psi_{\sigma s}=\psi_{\sigma s}(i_s),\quad \psi_{\sigma r}=\psi_{\sigma r}(i_r)$$

where:

$$i_\mu=\sqrt{\left(i_{sx}+i_{rx}\right)^2+\left(i_{sy}+i_{ry}\right)^2}\ ;$$

$$i_s=\sqrt{i_{sx}^2+i_{sy}^2}\ ;\qquad i_r=\sqrt{i_{rx}^2+i_{ry}^2}\ .$$

The currents of the rotor loops are defined as a sum of the currents of the *k* elements of the rotor bar.

$$i_{rx}=i_{r1x}+...+i_{rkx}\,;\qquad i_{ry}=i_{r1y}+...+i_{rky}\,.$$

The dynamics of the rotor is described by the equation

$$J\frac{d\omega_{м}}{dt}=M_e-M_c\left(t\right),\qquad (2)$$

where p_0 is the number of the pole pairs; *J* is the total torque of the inertia of the moving parts of the electric drive reduced to the IM shaft; $M_c\left(t\right)=M_c\left(t+T_M\right)$ is the loading diagram of the mechanism displaying the dependence on time *t* of the loading torque within the period T_M cycle; M_e is an electromagnetic torque of the motor

$$M_e=\frac{3}{2}p_0\left(\psi_{sx}i_{sy}-\psi_{sy}i_{sx}\right).$$

Let's move on from the angular velocity in equation (2) ω to slip *s*. As a result, we get

$$\frac{ds}{dt}=-\frac{3}{2}\frac{p_0^2}{J}\left(\psi_{sx}i_{sy}-\psi_{sy}i_{sx}\right)+\frac{p_0}{J}M_c(t).\qquad (3)$$

In a steady state mode at a periodic load torque variation of the flux linkage, the currents, of the rotor rotation speed, IM electromagnetic torque, etc., vary according to the periodic law. By calculating the periodic mode, the periodic dependencies and consequently, the average and root-mean-square values are determined.

To shorten the presentation of the algorithm to calculate the steady dynamic mode, the DE system consisting of equations (1) and (3) is written the form of the below vector equation

$$\frac{d\vec{y}(\vec{x},t)}{dt}+\vec{z}(\vec{y},\vec{x},t)=\vec{f}\left(\vec{u},M_c(t)\right),\qquad (4)$$

which:

$$\vec{y}=\boxed{\begin{array}{c}\vec{\psi}_{xy}\\ \hline s\end{array}}\ ;\quad \vec{x}=\boxed{\begin{array}{c}\vec{i}_{xy}\\ \hline s\end{array}}\ ;\quad \vec{z}=\begin{bmatrix}\Omega_{xy}\vec{\psi}_{xy}+R_{xy}\vec{i}_{xy}\\ -\frac{3}{2}\frac{p_0^2}{J}\left(\psi_{sx}i_{sy}-\psi_{sy}i_{sx}\right)\end{bmatrix};$$

$$\vec{f}=colon\left(U_m,0,...,0,p_0,M_c(t)/J\right);$$

$\vec{\psi}_{xy}$, $\vec{i}_{xy}$ – vectors of flux linkages and circuit currents;

$$\Omega_{xy}=\omega_0\times\begin{array}{|c|c|c|c|c|c|c|}\hline & -1 & & & \cdots & & \\ \hline 1 & & & & \cdots & & \\ \hline & & & -s & \cdots & & \\ \hline & & s & & \cdots & & \\ \hline \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \hline & & & & \cdots & & -s \\ \hline & & & & \cdots & s & \\ \hline\end{array}$$

The solution of the nonlinear system of equations (4) is a periodic dependency of vector components $\vec{x}(t)=\vec{x}(t+T)$. They can be determined by the evolutionary method (solving the Cauchy problem) by calculating the transient process under some initial conditions until the steady-state value which consists of repeating the process on adjacent periods. When the process is too slow, the curves at these periods differ little and the moment of the end of the transition process needs to be determined. The establishment method is practically unsuitable for calculating static characteristics.

The mathematical basis of the problem of calculating the coordinates of periodic dependencies during the period of the load torque variation is the theory of nonlinear oscillations [8]. It calculates periodic curves of the state variables as a pointwise boundary value problem with periodic boundary conditions. This result is thus obtained in a timeless domain, meaning that this is no need to calculate the transient process. In applied mathematics, the boundary value problems are considered for DE of the second and higher orders [9]. However, if the boundary conditions are given in a form of a ratio between the beginning and the end of the period, such a problem can be considered as a pointwise boundary value problem.

There are many methods available to solve boundary value problems. They differ in the way of algebraizing DE. One of them is the finite-difference method in which the derivatives are approximated by difference relations on the grid of nodes of the period according to one of the well-known formulas [9]. However, the system of algebraic equations that approximates DE on the period has a rather high order. The pointwise boundary value problem is solved by a projection method [10] which is based on the approximation of coordinates by splines of the third order [11]. The method is numerically stable and determines continuous dependencies of coordinates by calculating their discrete values at the *n* nodes of the grid per period. The transition from the DE system to its discrete analog follows formalized procedures.

$$H\vec{Y}(\vec{X})-\vec{Z}(\vec{Y},\vec{X})=\vec{B},\qquad (5)$$

where H is a block-diagonal matrix of size $n(3+2k)$ of the transition from a continuous change of coordinates to their nodal values, the elements of which are determined by a grid of nodes in period T_M [10];

$\vec{Y} = (\vec{y}_1, \ldots, \vec{y}_n)$, $\vec{Z} = (\vec{z}_1, \ldots, \vec{z}_n)$, $\vec{X} = (\vec{x}_1, \ldots, \vec{x}_n)$ – vectors composed of vector values $\vec{y}$, $\vec{x}$, $\vec{z}$ in the n nodes of the period.

The algebraic nonlinear system of equations (5) is a discrete analog of the DE system (4). Its unknown vector $\vec{X}$ with equation (5) is used, to construct periodic dependencies of each coordinate, including the electromagnetic torque, power, etc.

As a result of the change in the instantaneous value of the torque during period T_M, slip s oscillates relatively to the mean value. To take into account the deviation of the slip from the given constant at each node of the period, are refined the coordinates at the nodal points iteratively using the discrete equation for a steady press. In the paper, Newton's method is used [9] in which at the k-th step of the iteration the $(k+1)$-th approximation of the vector $\vec{X}$ is determined by the formula

$$\vec{X}^{(k+1)} = \vec{X}^{(k)} - \Delta\vec{X}^{(k)},$$

and $\Delta\vec{X}^{(k)}$ is determined from the system of equations

$$W^{(k)} \Delta\vec{X}^{(k)} = \vec{N}^{(k)}, \tag{6}$$

where:

$$\vec{N}^{(k)} = \begin{bmatrix} \Omega_{xy}^{(k)} \psi_{xy}^{(k)} + R_{xy} i_{xy}^{(k)} - \vec{u}_{xy} \\ \frac{3}{2} \frac{p_0^2}{J} \left(\psi_{sx}^{(k)} i_{sy}^{(k)} - \psi_{sy}^{(k)} i_{sx}^{(k)} - \frac{p_0}{J} M_c \right) \end{bmatrix}$$

Is a residual vector of the system (5);

$$W^{(k)} = \begin{bmatrix} L_{sxsx} - r_s & L_{sxsy} & L_{sxrx} & L_{sx1y} & \\ L_{sysx} & L_{sysy} - r_s & L_{syrx} & L_{syry} & \\ L_{rxsx} & L_{rxsy} & L_{rxrx} - r_r & L_{rxry} & -\psi_{ry} \\ L_{rysx} & L_{rysy} & L_{ryrx} & L_{ryry} - r_r & \psi_{rx} \\ -a_s \psi_{sy} & a_s \psi_{sx} & & & \end{bmatrix}$$

– the matrix of the Jacobian system (5) where the elements are the differential inductances of the IM circuits and flux linkages. As the matrix is written only for two rotor loops, marked with subscripts rx and ry and each divided into k elements the order of the matrix will in practice be greater.

Newton's iterative method has a quadratic convergence. It requires the formation of the initial values of the vector of the unknowns, which is located in the neighborhood of the method attraction. A reliable method to be used here is the method of continuation by parameter [12]. Since there are two perturbations in the system (5) of nonlinear algebraic equations i. e. the applied voltage vector and the load torque vector is $\vec{F} = \left(\vec{f}_1, \ldots, \vec{f}_n\right)$, the problem is solved in two stages. First, we increase the applied voltage, and then, assuming it is unchanged, we increase the nodal values of the applied torque. This makes it possible to determine the time dependencies of the coordinates in a steady-state periodic mode of the IM operation with a given law of the change of the applied torque which is refined by the iterative method.

The presented algorithm to calculate the steady-state is the basis for determining static characteristics obtained as a sequence of steady-states calculated for a number of coordinate values λ which is taken as an independent variable. In particular, it can be the torque of the inertia, the frequency or the relative pulse duration of the load torque period length, etc. Cyclic loading at a certain pulse frequency may give rise to mechanical resonance detectable by mathematical modeling.

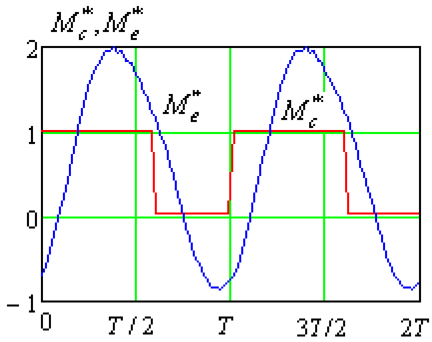
The problem of calculating static characteristics is solved using the differential method which differentiates the algebraic equation (5) with respect to an independent variable λ as a parameter. As a result, we get a nonlinear DE system of this argument. The static multidimensional characteristic is a dependence of periodic curves on independent variable λ obtained through the integration of the received DE system by λ. The initial conditions are obtained by performing the first step of the calculation at a particular supply voltage. At each integration step, the result can be refined using Newton's method. During the integration and iterative refinement, the circuit's differential inductances should be determined as the current's nonlinear functions. They are calculated with the IM mathematical model based on the above-mentioned magnetization curves according to [7].

Research results. Using the developed mathematical model and the presented algorithm to calculate the electric drive of the squirrel-cage induction motor, the steady-state modes, and static characteristics can be determined with any law of periodic variation of the load torque. The input data is the nonlinear characteristic of the magnetic-core magnetization by the main magnetic flux, the motor leakage fluxes, the winding data, and the specified cyclorama as a function of the load-torque time dependence.

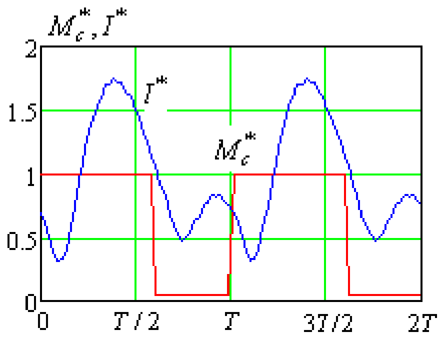
Calculation results of steady-state periodic modes and characteristics for the squirrel-cage induction motor 4AP160S4Y3 are P=15 kW, U=220 V and I = 29.9 A, p_0 = 2.

Besides the periodic time dependence of the coordinates at a given load, the mathematical modeling also detects with high reliability the possible resonant modes at certain inertia torques and cyclic load frequencies, thus order offering the possibility to avoid them. In practice, the mechanical resonance causes problems when IM operates at a frequency close to the one of the resonance, since the electromagnetic torque at the resonance frequency is zero (Fig. 2a). It is obvious that at the resonance point, the average value slip s for the period is also zero (Fig. 2b).

In certain operating states, the resonance effect may occur at a cyclically variable load. As the electromagnetic torque at the resonance frequency is zero, it is impossible to operate at a frequency close to the resonance. The IM resonance frequency depends on many factors, including the inertia torque. The resonance frequency cannot be determined in advance because the change in the load of the electric drive system completely changes the inertia torque. The presented mathematical model determines the resonance frequencies for a specific electric drive by changing the load variation frequency and calculating for each frequency the periodic torque dependence according to the presented algorithm, thus enabling the calculation of its average value for the period. The dependence of the average value of the electromagnetic torque on the frequency of pulses is thus obtained showing at which point of the dependence the torque decreases to zero. Fig. 3 shows examples of such curves. There may be more than one resonance mode (Fig. 2).

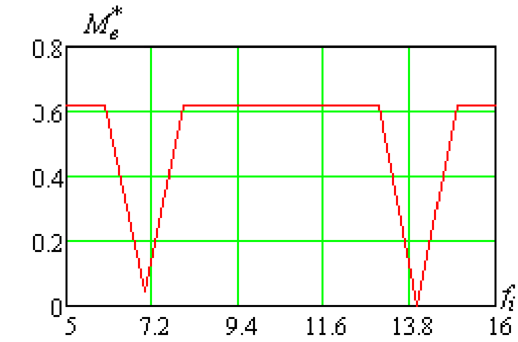


a)

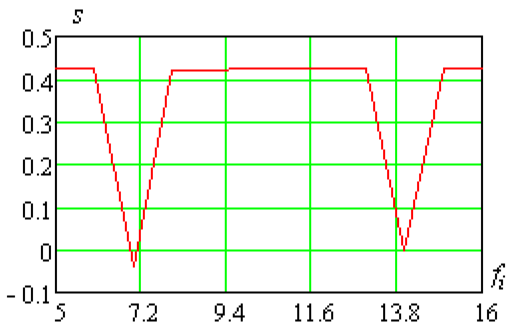


b)

Fig. 1. PerSiodic dependencies of the load torque (M_c^*), electromagnetic (M_e^*), and current (I^*) at the frequency

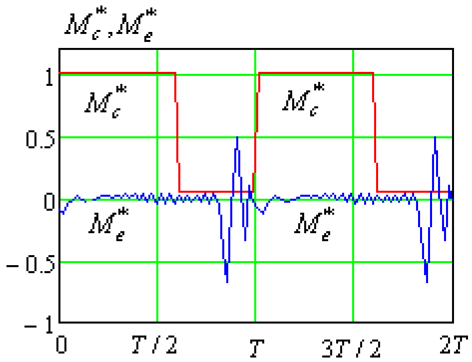


a)

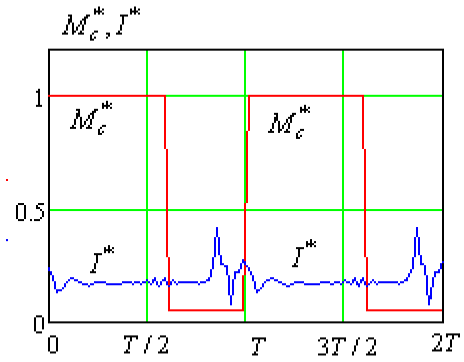


b)

Fig. 2. Dependencies of the average values of the electromagnetic torque (a) and slip (b) on the frequency impulse load



a)



b)

Fig. 3. Periodic dependences of the electromagnetic torque (a) and the effective value of the current (b) at a resonance occurrence

4 Conclusions

1. The operation of the squirrel-cage induction motor is analyzed by using mathematical modeling employing a mathematical model and corresponding algorithms and taking into account the saturation and currents displacement in the rotor bars at cyclically variable load states, and is investigated the frequency effect of variable load on the possibility of a mechanical resonance occurrence is investigated.
2. Using the presented algorithm to calculate steady-state periodic modes under cyclic loading solves the problem as a boundary value problem, which makes it possible to calculate a steady-state periodic mode in a timeless domain ensuring high performance and optimization calculations.
3. The developed mathematical model can be used to design and analyze the operation of electric drives at periodic and particularly cyclically variable loads. They provide a basis for developing an electric drive control system.

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