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Performance analysis of proportional navigation, neoclassical and pseudoclassical guidance methods on a 6DOF missile model

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Abstract. Proportional navigation (PN) is a missile guidance law usually used for low maneuvering targets. PN generates a commanding lateral acceleration normal to the Line of Sight (LOS) which commands the missile to steer toward a target. PN is determined by measauring the LOS angular velocity. For a maneuvering target, PN yields a non-zero final miss distance. The advanced guidance laws such as integrated guidance and control (IGC) or optimal missile guidance yield better results, but are much more difficult to implement. The paper presents variants of the traditional PN, which offer a similar result but require only the measurement of the LOS rate. The guidance laws are simualted on a 6DOF dynamic model and the results are compared with those obtained with the traditional PN guidance law.

Keywords: PN, Pseudoclassical navigation, Neoclassical navigation, 6DOF, Missile guidance.

Analiza učinkovitosti proporcionalne navigacije in metod vodenja na modelu izstrelka s šestimi stopnjami prostosti

Proporcionalna navigacija (PN) je način vodenja izstrelkov, ki se običajno uporablja za nizko manevrirajoče tarče. PN zagotavlja lateralno pospeševanje pravokotno na vidno črto (LOS), kar usmerja izstrelek proti cilju. PN temelji na meritvi kotne hitrosti LOS. Pri manevrirajoči tarči PN povzroči neničelni končni odmik od cilja. Napredni načini vodenja, kot sta integrirano vodenje in nadzor (IGC) ali optimalno vodenje izstrelkov, zagotavljajo boljše rezultate, vendar so bistveno bolj zahtevni za implementacijo. Prispevek predstavlja različice tradicionalne PN, ki ponujajo podobne rezultate, a zahtevajo le merjenje hitrosti spremembe LOS. Pravila vodenja smo simulirali na dinamičnem modelu izstrelka s šestimi stopnjami prostosti (6DOF). Rezultate smo primerjali z rezultati, pridobljenimi s tradicionalno metodo proporcionalne navigacije.

1 INTRODUCTION

Proportional navigation (PN) is perhaps the most used missile guidance method [1]. The main idea of PN is to generate lateral commanding accelerations which are normal to the instantaneous line of sight from a missile to the target. The intensity of the commanding lateral accelerations is proportional to the angular rate of the line of sight (LOS) angle. The commanding accelerations are forwarded to the autopilot subsystem which can be a linear or nonlinear controller [2, 3, 4] and has to make sure that the missile achieves the required accelerations determined by measring the LOS rate. If a missile achieves the required commanded accelerations and the target is not maneuvering, the target intercept is guaranteed. However, if the target is maneuvering then there might be some non-zero final miss distance. This can be solved by adding a correcting term to the commanded lateral accelerations. The term is proportional to the targets lateral acceleration. The approach is called an augmented proportional navigation [5]. As it is difficult to accurately estimate the target lateral accelerations. To remedy this, novel and more advanced missile guidance techniques are introduced such as integrated guidance and control [6, 7]. They can be based either on the optimal control [8, 9] or sliding mode control [10, 11, 12]. However, most of these new methods require measurement of the range, target velocity or LOS rate and are more difficult to implement then PN. Since PN is directly proportional to the LOS rate, it acts as a proportional controller in the guidance loop is given in the Figure 2. To improve steady-state performance and transient response, a PID controller is utilized. Though PID control is a very researched topic in the control system theory it had not been used in missile guidance algorithms before 2001 [13, 14]. The new approach is called the neoclassical missile guidance [13, 14]. It is a modification of the PN guidance law which delivers a zero miss distance (ZMD) when maneuvering with a deterministic or stochastic maneuver. It will be shown that the neoclassical guidance acts as a high-pass filter which differentiates the LOS rate. Its derivative is proportional to the targets lateral acceleration. Another [15] similar modification using feedforward and feedback control signals to make the real missile acceleration close to the commanded acceleration generated by the

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PN guidance law [15]. The approach is called pseudoclassical proportional navigation. The literature only considers the linearized guidance and missile dynamics. Our paper implements PN, neoclassical navigation and pseudo-classical navigation guidance methods on a 6DOF missile with a linear autopilot and a maneuvering target and compares these three types of the guidance laws. Our novelty of paper includes simulation of neoclassical and pseudoclassical missile guidance methods in three dimensions with a six-DOF missile model and a maneuvering target and a comparison between the PN, neoclassical and pseudoclassical missile guidance methods. This paper is organized as follows: Section 2 presents the missile 6-DOF model along with the aerodynamic coefficients used for its implementation. Section 3 explains the classical PN. Section 4 describes the missile autopilot used for three guidance methods. Section 5 shows actuator model. Section 6 explains the neoclassical missile guidance method and section 7 explains the pseudoclassical missile guidance method. Section 8 shows the target model with a lateral acceleration in three dimensions. Section 9 presents the Monte Carlo simulations for the range of target maneuvers and compares the three given algorithms, graphs and methods for the sinusoidal target maneuver. Section 10 draws the final conclusion.

2 Missile 6-DOF dynamic model

The missile motion is defined by six nonlinear differential equations of the first order. These equations are given by:

$$\dot{P} = L/I_x \tag{1}$$

$$\dot{Q} = PR\left(I_z - I_x\right)/I_y + M/I_y \tag{2}$$

$$\dot{R} = PQ\left(I_x - I_y\right)/I_z + N/I_z \tag{3}$$

$$\dot{u} = vR - wQ + F_x/m \tag{4}$$

$$\dot{v} = wp - uR + F_y/m \tag{5}$$

$$\dot{w} = uQ - vP + F_z/m \tag{6}$$

where u, v and w are the missile velocities along the x, y and z axes of the missile body frame. P, Q and R are the rotational angular velocities of the x, y and z axes of the missile body frame. L, M and N are are the moments acting along the x, y and z axes of the missile body frame. F_x, F_y and F_z are the total forces acting along the missile body axes. Forces acting along missile body axes are calculated as follows:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} + mg \begin{bmatrix} -\sin\theta \\ \sin\phi\cos\theta \\ \cos\phi\cos\theta \end{bmatrix} + \begin{bmatrix} F_{Ax} \\ F_{Ay} \\ F_{Az} \end{bmatrix} \tag{7}$$

where T is the thrust force, g is the gravitational acceleration and m is the mass of the missile, assumed to be constant. Angles θ, ϕ and ψ are the Euler RPY angles which the describe orientation in reference to the fixed inertial reference frame. Forces F_{Ax}, F_{Ay} and F_{Az} are the aerodynamic forces acting along the missile body axis. They are highly nonlinear functions depending on all the state variables and mostly upon the Mach number, angle of attack α and sideslip angle β. The angle of attack and the sideslip angle are calculated as follows:

$$\alpha = \arctan\frac{w}{v} \tag{8}$$

$$\beta = \arctan\frac{v}{u} \tag{9}$$

For a cruciform missile, these aerodynamic forces are calculated as follows [16]:

$$\begin{bmatrix} F_{Ax} \\ F_{Ay} \\ F_{Az} \end{bmatrix} = -qS \begin{bmatrix} C_{x_0} + C_{x_2}\left(\alpha^2 + \beta^2\right) \\ C_N\beta \\ C_N\alpha \end{bmatrix} \tag{10}$$

where S is the wingspan reference area, $q = 0.5\rho v_m^2$ is the dynamic pressure, $v_m = \sqrt{u^2 + v^2 + w^2}$ is the total missile velocity and ρ is the air density. Coefficients C_{x_0}, C_{x_2} and C_N are aerodynamic coefficients measured at different angles of the attack, sideslip angles and Mach number. These numbers can be determined in a wind tunnel or using a special software such as Missile DATCOM. For subsonic flights, these coefficients do not vary considerably. For a complete model, moments acting on the missile airframe need to be calculated. The aerodynamic forces act at the point called the center of pressure while the missile rotates around its center of gravity. These two points are displaced by distance r_x. Wing deflections and missile rotation also cause moments which need to be calculated. The total moments are calculated as follows [16]:

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} 0 \\ -r_X F_{AZ} \\ r_X F_{AY} \end{bmatrix} + \frac{qS}{v_m} \begin{bmatrix} C_{LP}P \\ C_{MQ}Q \\ C_{NR}R \end{bmatrix} + qS \begin{bmatrix} C_{L\delta_E}\delta_E \\ C_{M\delta_V}\delta_V \\ C_{N\delta_P}\delta_P \end{bmatrix} \tag{11}$$

where C_{LP}, C_{MQ} and C_{NR} are the aerodynamic stability coefficients, $C_{L\delta_E}$, $C_{M\delta_V}$ and $C_{N\delta_P}$ are the aerodynamic control coefficients and δ_E, δ_V and δ_P are the aileron, flaps and rudder deflections respectively. To define the whole model, a transformation from the angular velocities to the derivatives of the Euler RPY angles are defined as follows:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \tag{12}$$

Table 1 shows the used missile parameters [17] for the simulation inside Matlab and Simulink.

3 Proportional navigation

The PN generates commanding signals in terms of the lateral accelerations normal to the line of sight which are proportional to the angular rate of the line of sight [18]. Figure 1 shows the geometry of the PN in one guidance plane. The commanding lateral acceleration is given by:

I_x	I_y, I_z	m	C_{x_2}
$0.024\,\mathrm{kgm}^2$	$0.958\,\mathrm{kgm}^2$	$11.25\,\mathrm{kg}$	0.484
C_{x_0}	$C_{N\delta_P}$	$C_{M\delta_V}$	$C_{L\delta_E}$
2.04	0.0905	0.0905	0.0905
C_N	C_{MQ}	C_{NR}	C_{LP}
3.298	-10	-10	0.0905
T	r_x	ρ	S
$750\,\mathrm{N}$	$-0.119\,\mathrm{m}$	$1.225\,\mathrm{kg/m}^3$	$0.0314\,\mathrm{m}^2$

Table 1: Missile parameters.

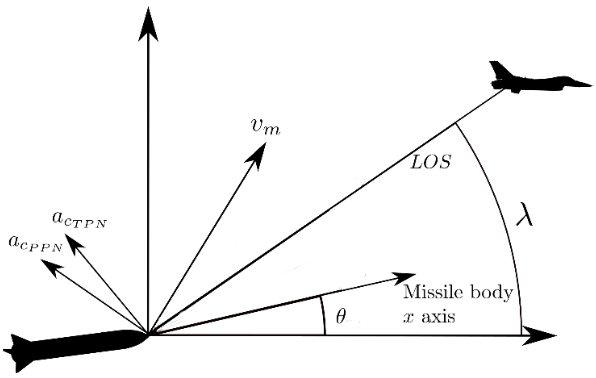

Figure 1: Geometry of the PN.

$$a_c = N v_c \dot{\lambda} \tag{13}$$

where λ is the line of the sight angle, v_c is the missile closing velocity and $N \geq 3$ is the effective navigation ratio.The LOS rate can be measured using infrared or electro-optical sensors. [19] uses machine learning to estimate the LOS rate and its PN usage. In order to calculate the closing velocity, the target velocity has to be known which requires the use of an active radar. [20, 21, 22] estimate the closing velocity based on the missile velocity to assume that the target is not moving and still achieving the zero miss distance. This PN variant is called the true PN [20]. It is given by:

$$a_c = N v_m \dot{\lambda} \tag{14}$$

In the three dimensions, the LOS rate is generalized as:

$$\omega = \dot{\lambda} = \frac{-r \times v_m}{r^2} \tag{15}$$

Therefore, the 3D true PN guidance law can be defined as:

$$a_c = N v_m \times \omega \tag{16}$$

The second element of vector a_c is the commanded lateral acceleration in the horizontal guidance plane and the third element is the commanded lateral acceleration in the vertical plane. The first element can be discarded. The guidance space can be divided into the horizontal and vertical guidance plane. The commanded lateral accelerations need to be generated for each of these planes.Assuming a small line of the sight angle, a linearized guidance model is obtained. Its relative distance y of the missile from the target is calculated using the following differential equation:

$$\ddot{y} = a_t - a_m \tag{17}$$

The model determines the LOS angle using the following expression:

$$\lambda = \frac{y}{v_c\,(t_f - t)} \tag{18}$$

where t_f is the total flight time and is assumed to be constant. The block diagram in Figure 2 shows the linearized guidance loop where $G_1(s)$ is the missile seeker dynamics modeled as a first order linear system and $G_2(s)$ is the missile dynamics. The Guidance loop

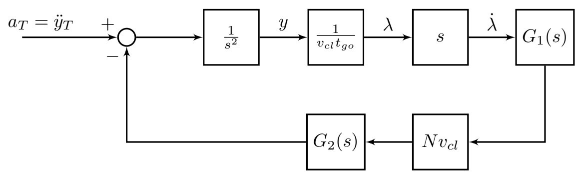

Figure 2: Linearized guidance loop.

dynamics is given as:

$$G(s) = G_1(s) G(s) \tag{19}$$

Despite being a a linearized model, it provides accurate results.

4 Autopilot design

The missile subsystem ensuring real missile accelerations equal to the command accelerations by the guidance law is referred to as the autopilot. When roll angle ϕ is zero, there will be no cross-coupling between the pitch and yaw channels of the motion. This is achieved using a PD controller with a measured roll angle in the feedback loop. The controller design is shown in Figure 3. The inner feedback loop is a damping loop which ensures minimal oscillations in the roll angle response. The outer loop ensures that the roll angle is always equal to zero. When the missile is stabilized in the roll channel, the pitch and the yaw movement are controlled independently. The main goal of the autopilot is to ensure that the missiles lateral acceleration is equal to the commanded lateral acceleration for te vertical and horizontal guidance plane. Therefore, there is one controller for the vertical plane and one for the horizontal guidance plane. The two controllers have the structure given in the Figure 3. The missiles lateral accelerations are given in the wind frame whose x axis is colinear with the wind direction. The derivative of the rotating velocity vector is given in body frame. The velocity derivative has to be transformed into the wind body frame via the following transformation matrix:

$$T_B^W = \begin{bmatrix} \cos\alpha\cos\beta & -\sin\beta & \sin\alpha\cos\beta \\ \sin\beta\cos\alpha & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{20}$$

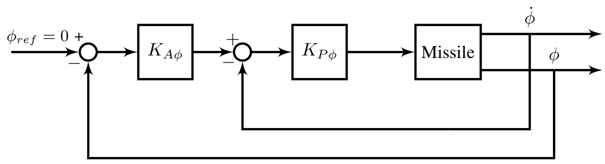

Figure 3: Roll controller design.

The missiles lateral accelerations are given by:

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = T_B^W \left(\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix} \right) \quad (21)$$

where a_y and a_z are the missile horizontal and lateral accelerations normal to the velocity vector in the wind frame and a_x is the acceleration along the velocity vector. As during the terminal guidance phase, the missile does not change its velocity. It is usually zero. Figure 4 shows the design of the lateral acceleration controller for vertical guidance plane. The aerodynamic

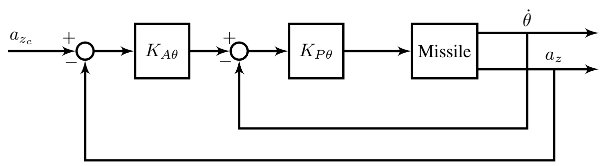

Figure 4: Lateral acceleration controller design.

coefficients change with respect to the Mach number, angle of attack, sideslip angle and other state variables. Therefore, it is advisable to create multiple controllers for various operating points. The design approach is called gain scheduling [4, 23]. Similarly as subsonic flights, these aerodynamic coefficients do not vary much, so it is enough to design only one controller.

5 Control surfaces and actuators

The presented missile dynamic model has three system inputs. The tactical missiles usually have the four control surfaces given in the $\times$ or $+$ configuration(see Figure 5). Combined motion of the four control surfaces gives either the elevator, rudder or aileron control deflections. In the $+$ configuration, surfaces 1 and 3 rotating in the same direction give rudder deflections. Similarly, surfaces 2 and 4 rotating in the same direction give elevator deflections. If surfaces 2 and 4 have an independent servo mechanism, their rotation gives the aileron deflections. In the $\times$ configuration, the autopilot pitch and yaw axes are each $45°$ from the planes of the adjacent control surfaces [16]. This implies that each of the four control surfaces is deflected equally for the both pitch and the yaw maneuvers. This means that the $\times$ configuration requires less surface deflections compared to the $+$ configuration. The autopilot calculates the elevator, rudder and aileron deflections. The computed

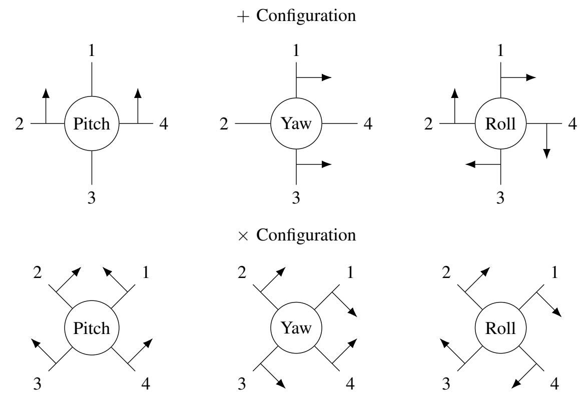

Figure 5: Missile wing configurations.

deflections are transformed into four wing deflections using the transformation matrix given by the following expression:

$$\begin{bmatrix} \delta C_1 \\ \delta C_2 \\ \delta C_3 \\ \delta C_4 \end{bmatrix} = \begin{bmatrix} -1 & \cos\phi_P & -sin\phi_p \\ -1 & \sin\phi_P & \cos\phi_p \\ 1 & \cos\phi_P & -\sin\phi_p \\ 1 & \sin\phi_P & \cos\phi_p \end{bmatrix} \begin{bmatrix} \delta P_{1C} \\ \delta P_{2C} \\ \delta P_{3C} \end{bmatrix} \quad (22)$$

where values δP_{1C}, δP_{2C} and δP_{3C} are the elevator, rudder and aileron commanding deflections calculated by the autopilot. Values δC_1, δC_2, δC_3 and δC_4 are the fin deflections required by each wing. The commanded fin deflections are usually limited. Angle ϕ_P is the angle by which the autopilot frame is rotated compared with the wings. For the angle $\phi_P = 0$, the missile is in the $+$ configuration while the angle $\phi_P = 45°$ yields the $\times$ configuration. The fins servomechanism ensures the fins to rotate for a commanded amount which yields real fin deflections δ_1, δ_2, δ_3 and δ_4. Fins are controlled by pneumatic, hydraulic and electric actuators depending on the maximum required fin moment. The fin transfer function is modeled as a second order transfer function:

$$G_{Fin}(s) = \frac{\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (23)$$

The fin servomechanism is implemented using a simple PID controller. Rreal fin deflections are then transformed back into the elevator, rudder and aileron deflections using the following transformation:

$$\begin{bmatrix} \delta_E \\ \delta_V \\ \delta_P \end{bmatrix} = \begin{bmatrix} -1/4 & -1/4 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix} \quad (24)$$

With the done transformations, the dynamic model is now more accurate since it takes into account also the saturation, dynamics and configuration of the actuators. The tactical missiles are considered in the $\times$ configuration.

6 Neoclassical missile guidance

The traditional approach to the missile guidance is to generate the commanding lateral acceleration proportional only to the LOS rate. It is known as the proportional navigation approach which produces miss distance in case of a maneuvering target. The modern approach is based on the optimal control which guarantees zero-miss-distance but needs much more details about the system and kinematic variables. The neoclassical guidance approach guarantees the zero-miss-distance (ZMD) as the other modern guidance laws, but it only requires measurements of the LOS rate. In [15, 18, 24], the miss distance is given by:

$$y(t_f) = \mathcal{L}^{-1}\left\{Q(s) y_T(s)\right\} \tag{25}$$

where:

$$Q(s) = e^{N\int_{\infty}^{s} H(\sigma) d\sigma} \tag{26}$$

$$H(s) = G(s)/s \tag{27}$$

$$y_T(s) = \mathcal{L}\left\{y_T(t)\right\} \tag{28}$$

where $\mathcal{L}$ is the Laplace transform and $y_T(t)$ is the deterministic input which can be either the target maneuver or the initial condition and $G(s)$ is the transfer function of the control dynamics and missile seeker assumed to be asymptotically stable. The zero miss distance distance is achieved at $Q(s) = 0$. Equation 26 can be written as:

$$Q(s) = e^{NF(s)}/e^{NF(\infty)} \tag{29}$$

where,

$$F(x) := \left[\int H(\sigma) d\sigma\right]_{\sigma = x} \tag{30}$$

For $Q(s)$ to be zero, $e^{NF(s)} \to \infty$ which requires $F(\infty) \to \infty$. Therefore, $H(s)$ needs to be determined for which $F(\infty) \to \infty$. Transfer function $H(s)$ is written as:

$$H(s) = \frac{b(s)}{a(s)} = \frac{b_1 s^{n-1} + b_2 s^{n-2} + \cdots + b_n}{s^n + a_1 s^{n-1} + \cdots + a_n} \tag{31}$$

When $r = \deg\left[a(s)\right] - \deg\left[b(s)\right]$ is relative order of transfer function $H(s)$, we get:

$$F(\infty) = \lim_{\sigma\to\infty} \int H(\sigma) d\sigma = \begin{cases} 0 & \text{if } r \geq 2 \\ \infty & \text{if } r = 1 \end{cases} \tag{32}$$

Since $H(s) = G(s)/s$, the interpretation of expression 32 is the following. If the dynamic of the guidance loop given by $G(s)$ is biproper, i.e. the degree of the numerator is the same as the degree of the denominator, the miss distance will be zero for each flight time and each deterministic or random target maneuver [13, 14, 25] In general, as $G(s)$ is a proper transfer function, a guidance controller composed of the lead networks should be added. However, since the pure lead controller can amplificy the noise, the lead-lag controller should be a variant of PN. It is called ZMP-PNG and is PNG with an additional lead controller. The guidance controller can be expressed in the following way:

$$K(s) = \prod_{i=1}^{r} \left(\tau_{Z_i} s + 1\right) \tag{33}$$

Since the PD controller can increase the noise, the following controller can be used:

$$K(s) = \prod_{i=1}^{r} \frac{\left(\tau_{Z_i} s + 1\right)}{\left(\tau_{P_i} s + 1\right)} \tag{34}$$

It should be noted that the former controller does not yield a proper guidance transfer function. Nonetheless, if the additional lag is not too large, a near ZMD can be achieved. Finally, a neoclassical ZMD-PNG commanded lateral acceleration is given by:

$$a_c = N v_m \prod_{i=1}^{r} \frac{\left(\tau_{Z_i} s + 1\right)}{\left(\tau_{P_i} s + 1\right)} \dot{\lambda} \tag{35}$$

It is shown that the second derivative of the LOS rate is directly proportional to the target lateral acceleration [18]. Therefore, the neoclassical high-pass filter acts as a LOS rate differentiator and acts in a sense equivalent of using both the LOS rate and the target acceleration for the commanded acceleration generation.

7 Pseudoclassical missile guidance

The PN guidance law is so popular that it is considered as classical [15]. The pseudoclassical missile guidance gives a modification of the classical PN guidance law which utilizes a classical control theory based on the feedback and feedforward control signals. As mentioned above, PN demonstrates a good performance against non-maneuvering targets. The modern guidance laws can produce good results but they require the target acceleration to be measured to obtain the required variables. The pseudoclassical guidance modifies the classical PN guidance law by using the measured real missile acceleration. The commanded acceleration generated by the pseudoclassical guidance is given by:

$$a_A = G_4(s) a_c + G_3(s)\left(a_c - a_M\right) \tag{36}$$

where a_M is the real missile lateral acceleration and a_c is the commanded lateral acceleration generated by PN or other similar guidance law. Transfer functions $G_3(s)$ and $G_4(s)$ are design parameters which guarantee the stability of the closed loop system.The pseudoclassical guidance law is given in Figure 6. The modified guidance law can be:

$$G_3(s) = \frac{k_1\left(\tau_{10} s + \mu\right)}{\tau_2 s + 1} \tag{37}$$

or

$$G_4(s) = k_2 \tag{38}$$

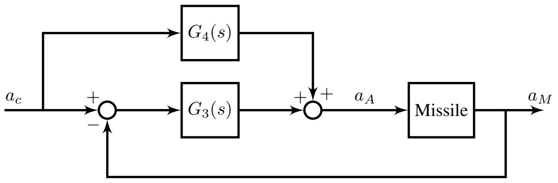

Figure 6: Modified guidance law.

8 Target model

In order to simulate and compare PN, neoclassical and pseudoclassical missile guidance, a kinematic target model is used. The guidance system disturbances are given as target lateral accelerations in the xz and xy inertial planes given by a_{Th} and a_{Tv}, respectively. They are normal to the targets velocity. The target velocity is given by its velocity in the horizontal (xy) and vertical (xz) plane as $v_{T_{xz}}$ and $v_{T_{yz}}$ respectively. Let x_T, y_T and z_T denote the targets relative positions in the inertial frame along the x, y and z axes, respectively. Let θ_h and θ_v be the targets velocity vectors orientations with respect to the x axis in horizontal and vertical plane respectively. Equations governing the target orientations are given by:

$$\dot{\theta}_h = a_{T_h}/v_{T_h} \tag{39}$$
$$\dot{\theta}_v = a_{T_v}/v_{T_v} \tag{40}$$

with given initial orientations θ_{h_0} and θ_{v_0}. The targets relative position in the inertial frame can now be defined by the following differential equations:

$$\dot{x}_T = v_{T_h}\cos\theta_h + v_{T_v}\cos\theta_v \tag{41}$$
$$\dot{y}_T = v_{T_h}\sin\theta_h \tag{42}$$
$$\dot{z}_T = v_{T_v}\sin\theta_v \tag{43}$$

for the given initial conditions x_{T_0}, y_{T_0} and z_{T_0}.

9 Simulation

Let us assume that the target is located at the initial distance of $1000m$ and at the height of $100\,m$ from the missile and let the targets total velocity in the xy plane be $30\,m/s$ and let the targets total velocity in the xz plane be $25\,m/s$ with zero initial headings in both planes. The initial RPY angles are $\psi_0 = \theta_0 = \phi_0 = 0$ which means that there is initial pitch heading error equal at $5.7°$ which has to be corrected. The initial missile velocitiy in the body frame is $\begin{bmatrix}100 & 0 & 0\end{bmatrix}^T\ m/s$. The maximum total missile velocity is $127\,m/s$. The initial target headings are $\theta_{h0} = \theta_{v0} = 0$. The neoclassical commanded acceleration is given with:

$$a_c = 5v_m\frac{0.203s^2 + 0.387s + 1}{0.0001s^3 + 0.038s^2 + 0.087s + 1}\dot{\lambda} \tag{44}$$

The pseudoclassical missile guidance law is defined as:

$$a_A = a_c + \frac{5}{0.25s + 1}(a_c - a_M) \tag{45}$$

Monte Carlo simulations are conducted for each of the guidance methods with the target horizontal lateral acceleration ranging from $0m/s^2$ to $4g$ and the target vertical lateral acceleration ranging between $-2g$ to $0m/s^2$. Figures 7, 8 and 9 graphically show the miss distance for each Monte Carlo run for each guidance law. The point at the origin indicates the zero miss distance. The average miss distances are shown in Table 2.

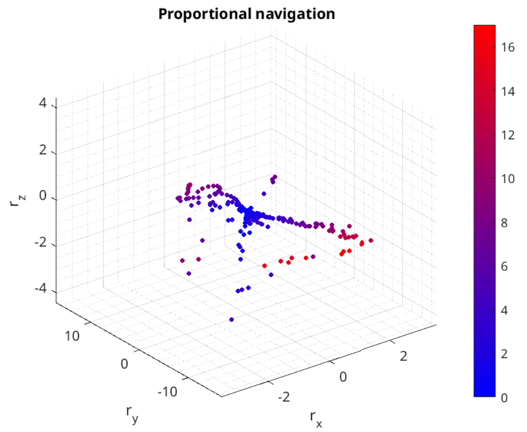

Figure 7: Monte Carlo simulations for PN.

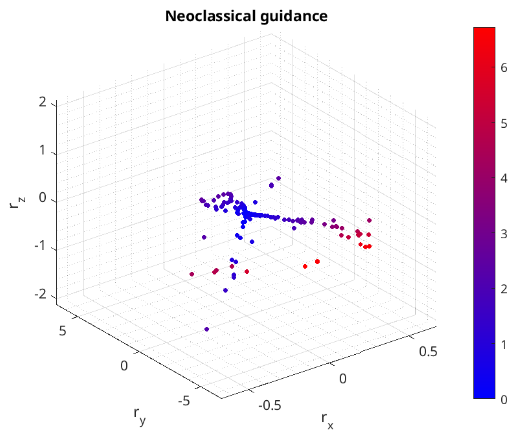

Figure 8: Monte Carlo simulations for the neoclassical guidance.

PN guidance average miss$[m]$	Neoclassical guidance average miss $[m]$	Pseudoclassical guidance average miss$[m]$
2.5637 m	0.6107 m	2.4534 m

Table 2: Average miss distance for each guidance method.

It can be seen that, on average, the neoclassical guidance gives the best mean final miss distance and a significant improvement compared to the PN guidance law. The pseudoclassical missile guidance law improves only the marginal final miss distance compared to the PN guidance law. Table 3 shows miss distances for these three types of the navigation. It can also be seen that

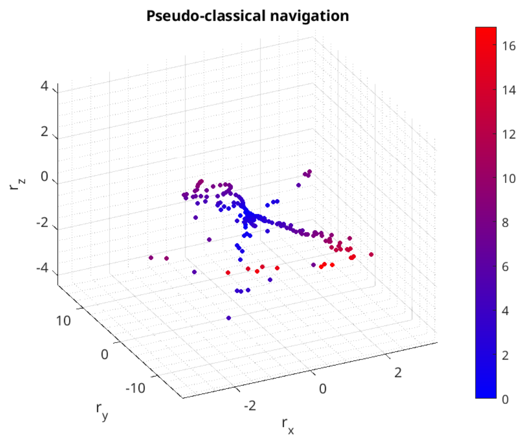

Figure 9: Monte Carlo simulations for the pseudoclassical guidance.

a_{T_V}	a_{T_H}	PN guidance miss	Neoclassical guidance miss [m]	Pseudoclassical guidance miss [m]
0	0	0	0	0
$-g$	g	0.05143	0	0.022096
$-g$	$2g$	0.013269	0	0
$-g$	$3g$	1.5325	0	0.74961
0	$4g$	14.853	6.3425	15.373
$-g$	$4g$	15.8815	6.2665	14.3231
$-2g$	g	4.4281	1.9184	4.4162
$-1.5g$	g	0	0	0
$-1.5g$	$1.5g$	3.0638	0.37573	2.5126

Table 3: Miss distances for various target maneuvers.

the neoclassical guidance gives the smallest miss distances, while the pseudoclassical navigation gives only a marginal improvement compared to the PN guidance. Even though the neoclassical guidance guarantees the zero miss distance when the total guidance transfer function is biproper which is not achieved here since in our case study the LOS angles are not small and the guidance loop is not linear. Furthermore, for some angles of the attack and sideslip angles, a higher order missile dynamics is introduced and the model confidelity is not of the second order. Let us now show the simulation where the target horizontal lateral acceleration is given by:

$$a_{Th} = 2g\sin(\omega t) \tag{46}$$

and the target vertical acceleration is:

$$a_{Tv} = -g\sin(\omega t) \tag{47}$$

where $\omega = 0.8\,\mathrm{rad/s}$ and both accelerations are normal to the total velocity vector and both of these acceleration vectors are normal to each other. Figure 10 shows that the missile intercepts the target with the neoclassical guidance. Similar missile paths are obtained in case of the PN and pseudoclassical guidance. Figures 11 and 12 show commanded lateral accelerations in the horizontal and vertical guidance plane respectively. It can be seen that the neoclassical guidance requires smaller missile accelerations compared to the PN and the pseudoclassical guidance.

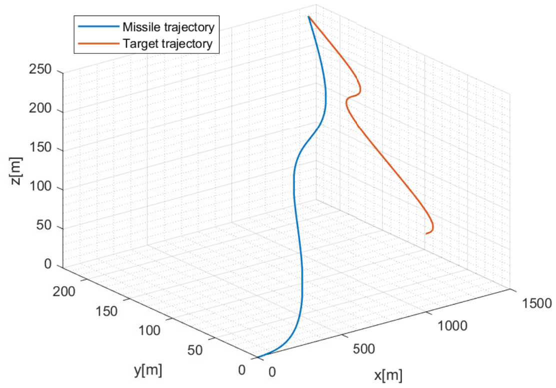

Figure 10: Missile and trajectory paths in case of sinusoidal target maneuvers.

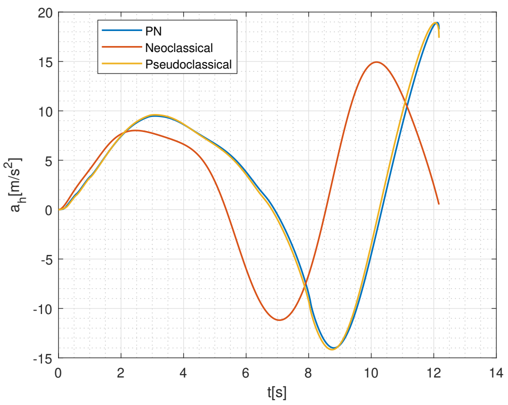

Figure 11: Commanding horizontal lateral accelerations.

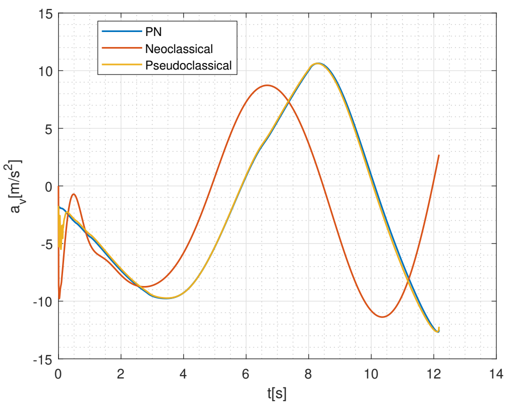

Figure 12: Commanding vertical lateral accelerations.

10 Conclusion

Th study briefly introduces PN and two PN variants called the neoclassical guidance and the pseudoclassical guidance. Each of these guidance methods requires only measurement of the LOS rate for their mechanization and implementation compared to the modern guidance laws which require much more information such as the range and closing velocity. Each of these guidance methods generates commanded lateral accelerations which are normal to the velocity vector (or LOS). A 6-DOF missile mathematical model is presented with aerodynamic coefficients. A wings and actuator dynamic model is also shown. A missile autopilot is presented. Its main idea is to ensure a zero roll angle which splits the guidance space into a vertical and horizontal plane. This is ensured via a roll PD controller with an inner damping loop. Two more controllers are used to ensure that the real missile accelerations are equal to the generated commanded accelerations. The Monte Carlo simulations show that the neoclassical guidance yields the smallest average final miss distance while the pseudoclassical guidance yields only a marginal improvement. It is also shown that the neoclassical guidance requires the smallest missile accelerations compared to the other two guidance methods.

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