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The Voltage Sags Matrix: a Useful Tool for Power Quality Analysis

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Abstract. The paper proposes a simple approximated method for calculation of voltage sags in an electrical network. Following the general method for symmetrical (three-phase) fault calculations based on the use of the bus impedance matrix of the network, the paper illustrates the construction of a square matrix that, under acceptable assumptions, directly provides voltage drops (sags) at each node of the network and for each faulted node. This matrix, which may be called the voltage sags matrix of the network, gives a synthetic representation of sags produced by symmetrical faults for the overall network. Approximations introduced are discussed and different possible applications of the matrix are outlined.

Key words: power quality, voltage sags, bus impedance matrix

Matrika padcev napetosti: Uporabno orodje za analizo kvalitete

Povzetek. Članek predstavlja preprosto približnostno metodo za izračun padcev napetosti v električnem omrežju. Na podlagi splošne metode za izračun simetričnih tripolnih kratkih stikov, ki temelji na impedančni matriki omrežja, opisuje delo oblikovanje kvadratne matrike za neposredno določanje padcev vozliščnih napetosti v omrežju. Matrika, imenovana matrika padcev napetosti, daje nazorno predstavo padcev napetosti zaradi simetričnih napak v celotnem omrežju. Delo opisuje in komentira poenostavitve metode in možnosti uporabe matrik v praksi.

Ključne besede: kvaliteta, padci anpetosti, vozliščna impedančna matrika

1 Introduction

Due to the wide diffusion of modern voltage sensitive equipment, power quality is today a big concern for several customers. Recent surveys clearly show that most industrial plants are more or less sensitive to voltage disturbances, and in particular voltage sags are often the most concerning ones, causing frequent protection tripping, incorrect load operation, interruption of processes and even the whole plant shut-down (see for instance [1]). Of course, even if interruptions are more severe disturbances than voltage sags, the latter actually are overall more dangerous because they are much more frequent than outages.

Short circuits in the electrical supply system are by far the most important source of voltage sags [2]. The main (even if not the only) characteristics of voltage sags are magnitude and duration. Prediction of voltage sag characteristics is a great opportunity to help system designers to select proper equipment specifications for critical processes, or to evaluate alternate configurations, or to compare different possible sites for future plants installation.

The sag duration depends on the clearing time of the overcurrent protective devices and then can be predicted starting from the fault clearing device characteristics. Prediction of the sag magnitude normally requires the preparation of an electrical model of the system, location of the fault and calculation of the voltage drop at any desired bus, as illustrated in [2]. Except for extremely simple systems (for instance small local radial distribution systems), this can be done using a computer program for network fault analysis. Several simulations are required to calculate voltage sags produced by any type of fault anywhere in the network.

The paper shows how, under reasonable assumptions, voltage sags produced by three-phase (symmetrical) faults can be immediately calculated starting from the bus impedance matrix of the net and generating a further matrix. This second matrix, which may be called the voltage sags matrix, directly provides voltage drops (sags) at each node of the net and for each fault node, giving so a synthetic representation of the sags produced by symmetrical faults for the overall network.

The construction of the voltage sags matrix requires the knowledge of the bus impedance matrix of the net. If the latter is not available, its building through the relevant algorithm requires the same electrical model of the system as the one required for computer simulation through

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a fault analysis program.

2 The voltage sags matrix

As well known, a linear electrical network can be represented in different ways. When bus currents and voltages are used, both the bus admittance matrix $|\mathbf{Y}|$ and impedance matrix $|\mathbf{Z}|$ can be used. The former is widely used in the power-flow analysis, while the latter is equally preferred in the fault analysis. For a given network, once the linear single-phase model is available, both matrices can be constructed through well known direct building algorithms that are straightforward processes on the computer. Therefore, in the following, we assume as available the net's bus impedance matrix. This is symmetrical (like the admittance matrix) and without zero-elements.

Let us follow the classical method for symmetrical fault calculations based on the bus impedance matrix, illustrated for instance in [3]. A three-phase fault at a generic bus 's' corresponds to a current injection $\mathbf{I}_{fs}$ at the same bus. The vector of the bus currents is then:

$$|\mathbf{I}_f| = \begin{vmatrix} 0 \\ 0 \\ . \\ -\ \mathbf{I}_{fs} \\ . \\ 0 \\ 0 \end{vmatrix} \tag{1}$$

If the voltages during the fault are represented by the vector $|\mathbf{U}_f|$ and the pre-fault voltages by the vector $|\mathbf{U}_o|$, we have:

$$|\mathbf{U}_f| - |\mathbf{U}_o| = |\mathbf{Z}| \cdot |\mathbf{I}_f| \tag{2}$$

The pre-fault voltage vector $|\mathbf{U}_o|$ is known, for any operative situation, for instance from a load-flow calculation. Expanding Eq. (2), we can write the voltage at bus 's' during the fault as:

$$\mathbf{U}_{fs} = \mathbf{U}_{os} - \mathbf{Z}_{ss} \cdot \mathbf{I}_{fs} \tag{3}$$

In case of a negligible fault impedance (as is usually the case) $U_{fs} \approx 0$ and the fault current is:

$$\mathbf{I}_{fs} = \frac{\mathbf{U}_{os}}{\mathbf{Z}_{ss}} \tag{4}$$

Considering the generic bus 'j', its during-fault voltage is:

$$\mathbf{U}_{fj} = \mathbf{U}_{oj} - \mathbf{Z}_{sj} \cdot \mathbf{I}_{fs} \tag{5}$$

and its voltage variation is given by:

$$\mathbf{U}_{fj} - \mathbf{U}_{oj} = \mathbf{\Delta U}_{fj} = -\frac{\mathbf{Z}_{sj}}{\mathbf{Z}_{ss}}\mathbf{U}_{os} \tag{6}$$

If all the elements of the impedance matrix have the same angle (for instance all reactance), the quantity -$\mathbf{Z}_{sj}/\mathbf{Z}_{ss}$ is a real negative number. This is a good assumption as far as high voltage systems (say over 50 kV) are concerned, where the reactance is by far prevailing and the resistance is often neglected. The approximation is not equally good when medium voltage (10-20 kV) distribution systems are included in the model. The X/R ratio of the HV/MV transformers is high, but medium voltage feeders have normally considerably lower X/R. Since these properties of the physical impedance of the net components are found also on the elements of the impedance matrix, the approximation appears acceptable whenever only the HV system is concerned or when the extension of the MV lines in the considered net is limited.

Under this hypothesis, Eq. (6) tells that the voltage variation $\Delta\mathbf{U}_{fj}$ at each generic bus 'j' is phase-opposite to the pre-fault voltage $\mathbf{U}_{os}$. Then, assuming $\mathbf{U}_{os}$ on the real axis, the situation might be for example like the one shown in Fig. 1, where the faulted bus 's' and the generic bus 'j' are outlined.

If the angles between the vectors $\mathbf{U}_{os}$ and $\mathbf{U}_{oj}$ are sufficiently small, the voltage change module $|\Delta\mathbf{U}_{fj}|$ can be confused, committing a negligible error, with the voltage drop or sag δ_j (note that the voltage sag magnitude δ_j indicates the voltage decrease, not the residual voltage). Working in p.u., and being $|\mathbf{U}_{os}|$ close to the nominal voltage U_n, this means:

$$\delta_j \cong |\mathbf{\Delta U}_{fj}|/U_n \cong |\mathbf{\Delta U}_{fj}|/U_{os} \tag{7}$$

The phase of the pre-fault voltages depends on the actual load-flow condition and cannot be predicted. In a large transmission system there is no assurance that the phase shift between the voltage vectors belonging to very far buses is small. However, since significant voltage sags at any given bus 'j' can be found when the faulted bus 's' is not electrically too far, the assumption that the angle between the vectors $\mathbf{U}_{os}$ and $\mathbf{U}_{oj}$ is sufficiently small appears fully acceptable.

From Eqs. (6) and (7) it follows that the voltage sag δ_{sj} at a bus 'j', caused by a fault in 's', is simply given by:

$$\delta_{sj} \cong Z_{sj}/Z_{ss} \tag{8}$$

Using Eq. (8) and letting $s = 1, \ldots, N$ and $j = 1, \ldots, N$ (being N the number of buses), it is immediately possible to build a matrix for the whole net (or for a part of it); we can call it the "voltage sags matrix".

The voltage sags matrix is no longer symmetrical, as clearly shown by Eq. (8). In this matrix, each column provides voltage sags on the corresponding bus produced

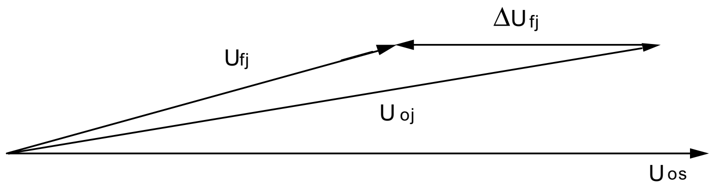

Figure 1. Pre-fault and during-fault voltage vectors at buses ‘s’ and ‘j

$$
\begin{array}{l}
\left|\begin{array}{cccccccc}
1 & \delta_{12} & . & . & \delta_{1j} & . & . & \delta_{1N} \\
\delta_{21} & 1 & . & . & \delta_{2j} & . & . & \delta_{2N} \\
. & . & 1 & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . \\
\delta_{j1} & \delta_{j2} & . & . & 1 & . & . & \delta_{jN} \\
. & . & . & . & . & 1 & . & . \\
. & . & . & . & . & . & 1 & . \\
\delta_{N1} & \delta_{N2} & . & . & \delta_{Nj} & . & . & 1
\end{array}\right|
\begin{array}{l}
\leftarrow\text{fault at bus 1} \\
\leftarrow\text{fault at bus 2} \\
\\
\\
\leftarrow\text{fault at bus j} \\
\\
\\
\leftarrow\text{fault at bus N}
\end{array}
\\
\begin{array}{cccccccc}
\uparrow & \uparrow & & & \uparrow & & & \uparrow \\
\text{Sag at} & \text{Sag at} & & & \text{Sag at} & & & \text{Sag at} \\
\text{bus 1} & \text{bus 2} & & & \text{bus } j & & & \text{bus } N
\end{array}
\end{array}
$$

Figure 2. Voltage sags matrix example for a N-bus net

by all different fault positions (i.e. changing the bus ‘s’), while each row provides voltage sags on all the buses produced by the corresponding (fixed) fault position (Fig. 2).

In the above, the pre-fault voltage magnitude at the faulted node U_{os} was assumed to be very close to the nominal value (1 p.u.). If the pre-fault voltage, known from load-flow calculations, is not sufficiently close to the nominal value and a more accurate calculation is required, the correction is immediate. For any fault bus operated below the nominal voltage ($U_{os} < 1$) the sags provided by the voltage sags matrix will be greater than the real ones and vice versa. For example, if U_{os} = 1.03 p.u., the voltage sags provided by the row ‘s’ should be corrected by a 3% increase and so on.

3 Applications of the voltage sags matrix

Once the bus impedance matrix of a given net is known or has been calculated, the voltage sags matrix is very easy to build, even by hand calculations. The matrix directly provides approximated voltage sags at each bus corresponding to symmetrical faults at each net node. The approximation consists in the confusion between the module of the voltage change vector caused by the fault and the voltage sag magnitude, and it is fully acceptable whenever all the elements of the impedance matrix have roughly the same angle and the pre-fault voltages have a limited phase-difference.

The voltage sags matrix can be used for sags prediction, providing an alternative and often easier way to computer simulation. The limit of this “fast” approach, compared to computer simulation, is that the voltage sags matrix, defined through Eq. (8), accounts for only three-phase (symmetrical) faults. Three-phase faults are not the most frequent but they are almost always the most severe faults, causing the deepest sags compared to asymmetrical (phase-to-ground, phase-to-phase or two-phases-to-ground) faults, as reported for example in [2]. Then, it can be stated that the voltage sags matrix provides with sufficient accuracy the most severe sags produced at each net node by faults anywhere in the net.

For any given location, the matrix allows an immediate determination of the area of the net (that may be called the area of vulnerability) where faults cause a voltage below any set limit. If the considered location is the generic bus ‘j’, inspection of column ‘j’ provides the required area for any given set limit.

If the considered location does not coincide with a net node, or if a refinement in the definition of the area of vulnerability is required (for instance to individuate the part of a transmission line that is included in the area), new artificial buses have to be included along the lines. Accordingly, the bus impedance matrix of the net can be easily modified through proper use of its building rules.

The calculation of the voltage sags matrix follows immediately.

Additionally, if the fault rates (expressed for example in faults/year) of the different components of the net (lines and transformers) are available, the individuation of the area of vulnerability relevant to any given bus allows an easy calculation of the expected number of voltage sags exceeding the equipment sensitivity limit. This approach can be very useful for customers sensitive to voltage sags when:

- purchasing voltage-sensitive equipment (with a given sensitivity limit)
- investigating desensitising remedial actions
- forecasting effects (process shut-down, costs) related to voltage sags.

Note that several variables must be considered for an accurate assessment of voltage sags, including all types of fault, positive, negative and zero sequence network modeling, fault impedance and actual pre-fault voltages [2]. This entails a considerable work, but the accuracy is likely to be frustrated by the large uncertainties often existing on the actual fault rates of the net components. It appears then practical and reasonable to give up a great accuracy and to use simple approximated methods.

A further application of the voltage sags matrix concerns the evaluation of the independence between any couple of buses in a given net. For instance, the bus independence evaluation is required when designing a Static Transfer Switch (STS) for power quality improvement at a given site through fast load transfer. In this case, a proper selection (or planning) of a secondary independent voltage source is the basic prerequisite for an effective STS operation [4].

More generally, since the voltage sags matrix can be used for voltage sag assessment purposes at any bus in the net, a further application concerns the individuation of suitable locations (from the power quality point of view) for installation of voltage sensitive customer plants.

4 Conclusion

Starting from the bus impedance matrix of the network, the paper proposes an approximated method for immediate calculation of voltage sags caused by three-phase symmetrical faults. The voltage sags matrix directly provides sags at each node of the network due to faults taking place anywhere in the network, thus representing a meaningful tool for power quality analysis in the overall network. In particular, this method could be conveniently used for:

- voltage sags prediction
- determination of the areas of vulnerability and voltage sags assessment
- evaluation of nodal independence
- individuation of suitable sites (from the power quality point of view) for future plant installation.

5 References

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