

Implementation of a method of numerical integration of average voltages at the integration step for designing microcontroller models of electrotechnical systems

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Abstract. Hardware implementation of models of electrotechnical and electromechanical systems is widely recognized as a practice allowing hardware-in-the-loop technique, as well as simpler techniques like state observers or estimators. While hardware-in-the-loop technique is rather complicated, state observers or estimators are often implemented even on low-end microcontrollers. The paper presents a method of numerical integration of average voltages at the integration step for implementing a model of electrotechnical system on a low-end microcontroller because of its high operational speed and numerical stability. The developed model is emulated as a program in the C language and implemented on the Texas Instruments MSP430 platform. General guidelines are proposed for implementing of a method for designing a microcontroller model of the electrotechnical system. Such model is suitable as a basis for designing microcontroller emulators of electrotechnical and electromechanical systems or their elements serving as estimators or state observers.

Keywords: electrotechnical system, microcontroller model, emulation, low-end microcontroller, method of average voltages at the integration step

Izvedba numerične integracije povprečnih vrednosti napetosti v integracijskem koraku pri snovanju mikrokrmilniških modelov elektrotehniških sistemov

Za modeliranje elektrotehniških in elektromehanskih sistemov s strojno opremo uporabljamo strojno opremo v zanki in opazovalnike stanj. V primerjavi s strojno opremo v zanki lahko opazovalnike stanj izvedemo z mikrokrmilniki. V članku je predstavljena metoda za numerično integracijo povprečnih vrednosti napetosti v integracijskem koraku pri izvedbi modela elektrotehniškega sistema z mikrokrmilnikom. Razviti model smo emulirali v programskem jeziku C in izvedli na platformi Texas Instruments MSP430. Predlagamo splošne smernice za načrtovanje elektrotehniškega sistema, izvedenega z mikrokrmilniškim modelom. Predstavljeni model je primeren kot osnova za načrtovanje mikrokrmilniških emulatorjev elektrotehniških in elektromehanskih sistemov.

1 INTRODUCTION

The simulation technology enhances its functionality by enabling operation of virtual models together with the physical equipment via ADC and DAC.

One of the techniques enabling this operation is hardware-in-the-loop (HiL), where the simulation software replaces a process or an object as a part of a complicated system. Examples of the HiL applications are, but not limited to: testing of controllers [1], failure analysis of the power equipment [2], *etc.* These applications require a real-time operation for their functioning. Thus, a suitable resolution for a 50-Hz power system is roughly 50 μ s; high-frequency switching circuits require smaller steps [3]. Hence, the technologies such as FPGA ([1, 2]) or DSP ([4]) are utilized as a hardware basis.

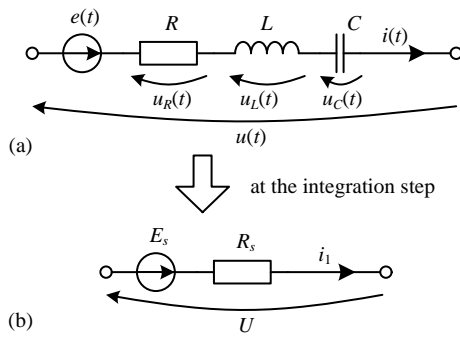


Figure 1. Branch of the electric circuit (a) and its equivalent circuit at the integration step according to AVIS (b).

Other techniques employing a hardware implementation of models are state observers or estimators. They are softwares operating in parallel with a physical equipment that estimates an inner state or inner variables of a real system. Compared to HiL, in general, they require simpler models, depending on the complexity of the system description, e.g., the model of a wind turbine may be used for estimating the wind speed [5], or the wind speed and a couple of electrical variables [6], such as active powers, currents *etc.* Observers or estimators are implemented even on low-end microcontrollers [7, 8]; however, this often makes the real-time operation impossible.

Recent paers are devoted to implementing models of electrical and mechanical systems [9], PEM fuel cell [10], varicap diode [11], *etc.*, on low-end microcontrollers. However, these models are suitable for applications with slow-varying values or even for those operating with steady-state values. However, even low-end microcontrollers are utilized as a tool for testing the design methodologies used for implementation of a model on a microcontroller [9].

Communications of models with a physical equipment are widely utilized in laboratory teaching of students as demonstrated on examples of Universidad de los Andes (Bogotá, Colombia) [12], National Technical University of Athens (Greece) [13], and the Federal University of Technology (Paraná, Brazil) [14].

In the paper we propose to utilize a method of average voltages on the integration step (AVIS) [15] for the mathematical description of electrotechnical systems due to its high operating speed and numerical stability as compared to conventional methods of numerical integration for electrotechnical [16] and electromechanical [17, 18] systems. The paper introduces the AVIS method for implementing models of electrotechnical systems on a low-end microcontroller.

2 ANALYSIS OF ELECTRIC CIRCUITS USING THE AVIS METHOD

2.1 General Description of the Method

In most common cases, every single branch of a complex electric circuit is represented as serial-connected electromotive force e , resistance R , inductance L , and capacitance C with applied voltage u on its terminals (Fig. 1a).

Considering integration step Δt starting at t_0 , the Kirchoff's voltage law for the branch is represented in the form of formula (1):

$$\frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} (u - e - u_R - u_C - u_L) dt = 0, \quad (1)$$

where u , e , u_R , u_C , and u_L are instantaneous values of the applied voltage, electromotive force, and voltages on appropriate elements of the branch. In other words, equation (1) represents the AVIS balance for the branch.

The AVIS method is based on algebraization of differential equations representing the branch, considering the current waveform at the integration step between i_0 and i_1 as an m -order polynomial [15]. The consideration results in representing the branch in Fig. 1a at the integration step as serial-connected equivalent electromotive force E_s and equivalent resistance R_s with a current i_1 flowing through the branch (Fig. 1b). Therefore, E_s and R_s are calculated at each integration step as follows [15]:

$$E_s = E - u_{R0} - u_{C0} + \left(\frac{R}{m+1} + \frac{\Delta t}{C} \cdot \frac{2 - (m+1)(m+2)}{2(m+1)(m+2)} + \frac{L_0}{\Delta t} \right) i_0 - \left(\frac{R\Delta t^k}{(k+1)!} \cdot \frac{m-k}{m+1} + \frac{\Delta t^{k+1}}{C(k+2)!} \cdot \frac{(m+1)(m+2) - (k+1)(k+2)}{(m+1)(m+2)} \right) \frac{d^{(k)} i_0}{dt^{(k)}} \quad (2)$$

and

$$R_s = \frac{R}{m+1} + \frac{\Delta t}{C(m+1)(m+2)} + \frac{L_1}{\Delta t}, \quad (3)$$

where i_0 is the current, u_{R0} and u_{C0} are the voltages at the resistance and capacitance, all at the beginning of the step; L_0 and L_1 are the branch inductances at the beginning and at the end of the step; U and E are the

average values at the step of the applied voltage and

electromotive force: $U = \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} u dt$, and $E = \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} e dt$.

2.2 Algorithm for the Electric-Circuit Analysis at the Integration Step

Analysis of the electric circuit using the AVIS method at each integration step [16] is further explained. First, the equivalent diagram at the integration step for the analyzed circuit is generated (an example of the electric circuit and its equivalent diagram at the integration step is demonstrated in Fig. 2). Afterwards, the following is performed at the integration step:

1. Calculation of derivatives at the beginning of the integration step. Equations are based on the known methods to analyze the electric circuit.

2. Determination of the equivalent parameters of every branch of the electric circuit (equivalent electromotive force E_{si} (2) and resistance R_{si} (3)).

3. Calculation of the currents at the end of the integration step. Since the circuit at the integration step is treated as a DC circuit, the calculation can be based on the Kirchhoff's law, loop current method or nodal analysis.

4. Determination of the voltages on the capacitor at the end of integration step u_{C1} . Equation (4) is used [16]:

$$u_{C1} = u_{C0} + \frac{\Delta t}{C(m+1)} (m \cdot i_0 + i_1) + \sum_{k=1}^{m-1} \frac{\Delta t^{k+1}}{(k+1)!} \cdot \frac{1}{C} \cdot \frac{m-k}{m+1} \cdot \frac{d^{(k)} i_0}{dt^{(k)}}. \quad (4)$$

2.3 Example of the Electric-Circuit Analysis at the Integration Step

The circuit of Fig. 2a is depicted by mathematical model ((5)-(8)). These formulae are an application of (2)-(4) for each branch of the circuit utilizing the second-order AVIS ($m = 2$). Thus, the first derivatives of the currents at the beginning of the integration step are defined as:

$$\begin{aligned} \frac{di_{10}}{dt} &= \frac{E_1 - R_1 i_{10} - R_3 i_{30} - u_{C30} - R_4 i_{40}}{L_1}, \\ \frac{di_{20}}{dt} &= \frac{u_{C30} - u_{C20} + R_3 i_{30} + R_4 i_{40}}{L_2}, \\ \frac{di_{50}}{dt} &= \frac{E_5 - R_5 i_{50} - R_4 i_{40}}{L_5}, \\ \frac{di_{30}}{dt} &= \frac{di_{10}}{dt} - \frac{di_{20}}{dt}, \\ \frac{di_{40}}{dt} &= \frac{di_{30}}{dt} + \frac{di_{50}}{dt}. \end{aligned} \quad (5)$$

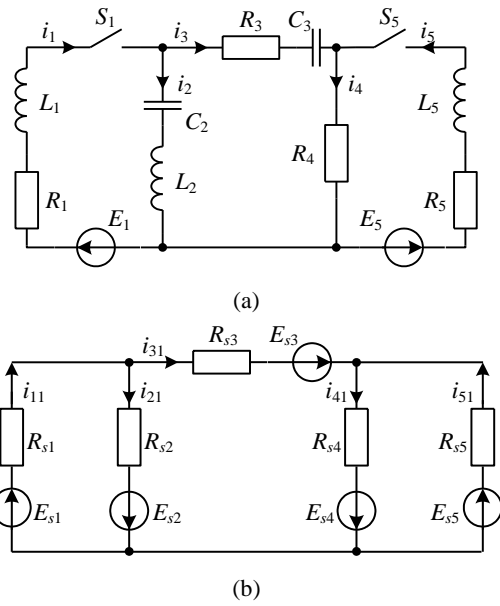


Figure 2. Diagram of the analyzed electric circuit (a) and its equivalent diagram at the integration step using AVIS.

The equivalent electromotive forces of the branches are:

$$\begin{aligned} E_{s1} &= E_1 - R_1 i_{10} + \left(\frac{R_1}{3} + \frac{L_1}{\Delta t} \right) i_{10} - \frac{R_1 \Delta t}{6} \frac{di_{10}}{dt}, \\ E_{s2} &= -u_{C20} + \left(\frac{L_2}{\Delta t} - \frac{5\Delta t}{12C_2} \right) i_{20} - \frac{\Delta t^2}{12C_2} \frac{di_{20}}{dt}, \\ E_{s3} &= -R_3 i_{30} - u_{C30} + \left(\frac{R_3}{3} - \frac{5\Delta t}{12C_3} \right) i_{30} - \\ &\quad - \left(\frac{R_3 \Delta t}{6} + \frac{\Delta t^2}{12C_3} \right) \frac{di_{30}}{dt}, \\ E_{s4} &= -R_4 i_{40} + \frac{R_4}{3} i_{40} - \frac{R_4 \Delta t}{6} \frac{di_{40}}{dt}, \\ E_{s5} &= E_5 - R_5 i_{50} + \left(\frac{R_5}{3} + \frac{L_5}{\Delta t} \right) i_{50} - \frac{R_5 \Delta t}{6} \frac{di_{50}}{dt}; \end{aligned} \quad (6)$$

the equivalent resistances of the branches are:

$$\begin{aligned} R_{s1} &= \frac{R_1}{3} + \frac{L_1}{\Delta t}, \\ R_{s2} &= \frac{\Delta t}{12C_2} + \frac{L_2}{\Delta t}, \\ R_{s3} &= \frac{R_3}{3} + \frac{\Delta t}{12C_3}, \\ R_{s4} &= \frac{R_4}{3}, \\ R_{s5} &= \frac{R_5}{3} + \frac{L_5}{\Delta t}; \end{aligned} \quad (7)$$

and finally, the voltages on the capacitors at the end of the integration step are calculated as:

$$\begin{aligned} u_{c21} &= u_{c20} + \frac{\Delta t}{3C_2}(2i_{20} + i_{21}) + \frac{\Delta t^2}{6C_2} \frac{di_{20}}{dt}, \\ u_{c31} &= u_{c30} + \frac{\Delta t}{3C_3}(2i_{30} + i_{31}) + \frac{\Delta t^2}{6C_3} \frac{di_{30}}{dt}. \end{aligned} \quad (8)$$

3 IMPLEMENTATION OF THE MATHEMATICAL MODEL ON A MICROCONTROLLER

3.1 Parameters of the Selected Platform

The main objective of our study was to implement AVIS method on a low-cost microcontroller platform and to observe any possible “bottlenecks” of the implementation. Therefore, the selected platform is a 16-bit Texas Instruments MSP430G2553 with the clock speed up to 16 MHz, 512 B RAM, and 2×16 -bit timers.

Before implementing the model, we should note that the typical applications of the MSP430 family include “low-cost sensor systems that capture analog signals, convert them to digital values and then process the data for display or transmission to a host system” [19]. However, the platform provides a tool for testing the design as well as for debugging the implementation algorithm.

In contrast to [9], the selected platform allows for a floating-point operation, for which reason there are no rounding errors for the fixed-point operation.

3.2 Implementing AVIS on a Microcontroller

Consider an electrotechnical system described by the connections such as the electric circuit in Fig. 2a with switches S_1 and S_2 turning on simultaneously. Then, the mathematical model of the system using the AVIS method is described by (5)-(8), and the algorithm of the analysis of the system would be as described in Subection 2.2. The currents at the end of the integration step and the voltages on the capacitors are unknown; the currents are calculated by the loop-current method by using the Cramer’s rule. These calculations are performed in a loop and implemented on a microcontroller as a C program utilizing the IAR Embedded Workbench.

As for the authors’ opinion, the simplest technique of observation of the calculated variables is setting the dependence between the variables and duty cycle of the output PWM signal [20]. The microcontroller timer utilizes the SMCLK clock (of about 1.1 MHz) and sets the PWM period (Capture Compare Register 0, CCR0) to 256 clock ticks (that gives about 4 kHz). The timer runs in the “up” mode, counting from zero to the CCR0 value. Capture Compare Register 1, CCR1, is used for setting the duty cycle (when the timer counts to the

CCR1 value, an interrupt is generated). Afterwards, the RC low-pass filter converts the obtained PWM signal to an analog waveform.

The time scale for each waveform is expanded so as to obtain one iteration result for each change in the PWM period. Therefore, the waveforms are suitable for observation on an oscilloscope. After iteration, any further calculation stops until the permission (until the key switches from 0 to 1). Afterwards, the final values of the currents or voltages are converted to integer values and resized to match the desired range from 0 to 256 (in a real application, the range is closer, e.g. from 50 to 250 in order to avoid distortions).

At the interrupt, the calculated integer value defines the CCR1 value and permits further calculations (the key switches to 1).

4 RESULTS AND DISCUSSIONS

As noted, when the algorithm is implemented on a microcontroller platform, the iteration time is 41.4 ms. At a time step of 10^{-4} s, this gives only 2.4 ms of the emulated time (the time taken for a model emulated on a microcontroller) for a second of a real time.

When the parameters of the circuit are constant, the equivalent resistances at step R_{si} are constant, too (3). Therefore, with a simplified algorithm, the calculation made outside the loop, the time taken for the iteration is hence reduced to 37.4 ms. Another computation optimization involves a matrix operation. Using specific formulae to calculate the currents at the end of a simulated integration step, instead of solving the matrix equations by using appropriate functions in a general form, reduces the iteration to 25.2 ms. This makes the designed emulator suitable for only a non-real-time operation when the steady-state values are required.

Fig. 3 demonstrates results of using a mathematical model of a system on a microcontroller. Each subfigure contains two plots; one corresponds to a desktop simulation (*simulated*), and the other is a waveform obtained after filtering the output PWM signal of a microcontroller (further termed *measured*) as explained in Subsection 3.2. Both plots are combined into a single graph so that the real time of the *measured* plot corresponds to the calculation time of the *simulated* one. The time ratio, and also the scales on both ordinate axes, are selected so that the measured value and the anticipated value of the measurement can be compared at any time.

Fig. 3 represents the values that correspond to the currents and voltages in the electrotechnical system in Fig. 2a. Compared to the desktop simulation, emulation on a microcontroller provides an adequate time-scalable representation of the simulated signals.

5 CONCLUSIONS AND FUTURE WORK

The method of numerical integration of average voltages at the integration step (AVIS) is implemented

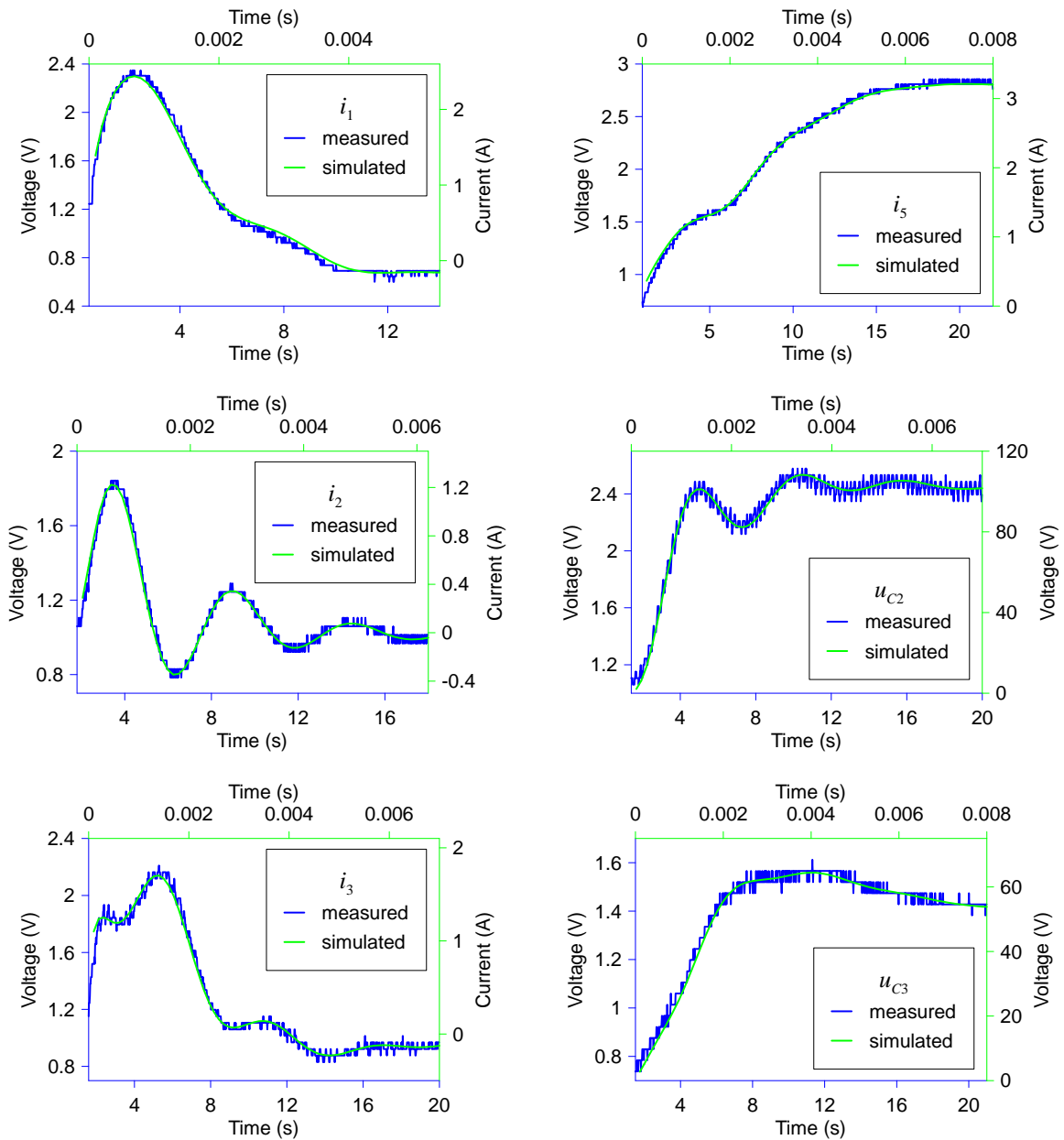


Figure 3. Waveforms of the values of the circuit in Fig. 2a as a result of a desktop simulation and emulation on a microcontroller (voltages measured at the output of the microcontroller).

on a low-end microcontroller platform to design microcontroller models of electrotechnical systems. The method is selected and used for its high operational speed and numerical stability.

Our model of the electrotechnical system utilizing the AVIS method is realized as a program in the C language and implemented on a Texas Instruments MSP430 platform. As shown, the device emulates the system, thus, computational capacities of the platform allow using it as a base for the state observers or estimators.

At the current stage, our study serves only as a general guideline for implementing a method of the average voltages at the integration step for designing

microcontroller models of electrotechnical systems. Our future work will be towards: (i) selection of a practical for emulators, such as electric drives, photovoltaic systems, *etc.*; (ii) implementation of guidelines for developing low-cost platform real-time applications.

APPENDIX A PARAMETERS OF THE SIMULATED CIRCUIT

$$\begin{aligned}
 E_1 &= 100 \text{ V}, R_1 = 20 \text{ } \Omega, L_1 = 10 \text{ mH}, \\
 L_2 &= 10 \text{ mH}, C_2 = 10 \text{ } \mu\text{F}, \\
 R_3 &= 1 \text{ } \Omega, C_3 = 10 \text{ } \mu\text{F}, R_4 = 15 \text{ } \Omega, \\
 E_5 &= 50 \text{ V}, R_5 = 1 \text{ } \Omega, L_5 = 20 \text{ mH}.
 \end{aligned}$$

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