

# Design of an Anti-aliasing Active Filter for a Hydroacoustic Receiving Station Using Discrete Non-linear Mathematical Optimization

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**Abstract.** The paper deals with designing an active polynomial filter using a search method of non-linear mathematical programming in a discrete parameter space. When designing, models of real operational amplifiers are taken into account by macromodelling in a frequency domain. A synthesis of an active anti-aliasing filter for a hydroacoustic receiving station is given as an example. The filter theoretical and experimental characteristics are analyzed.

**Keywords:** filter design, aliasing, mathematical optimization.

## Snovanje sita proti prekrivanju spektra za hidro-akustično sprejemno postajo z uporabo diskretne nelinearne matematične optimizacije

V prispevku smo predstavili snovanje sita proti prekrivanju spektra z uporabo diskretne nelinearne matematične optimizacije. Pri načrtovanju sita smo uporabili modele operacijskih ojačevalnikov z makromodeliranjem v frekvenčnem območju. Kot primer uporabe smo pokazali sintezo sita za hidroakustično sprejemno postajo. Analizirali smo teoretično izračunane in eksperimentalno izmerjene karakteristike sita.

## 1 INTRODUCTION

Digital processing of hydroacoustic signals involves conversion of an analog signal to a digital one using an analog-to-digital converter (ADC). When discretizing a signal, false signals may appear due to aliasing that overlaps of discrete signal spectra [1]. To prevent aliasing, which leads to signal distortion, it is necessary to limit the signal spectrum before digitization by an analog low-pass filter. The cutoff frequency of such an anti-aliasing filter (AAF) is set equal to the Nyquist frequency, i.e. half the sampling frequency ( $f_s/2$ ). Phase-modulated signals are most often used in the underwater communication systems. However, analysis of the spectrum at the entry of a hydroacoustic receiving station shows that there are noise signals in the low-frequency range (up to 50-100 Hz), where the power may exceed the level of a useful hydroacoustic signal by an order of magnitude (Fig. 1).

The presence of such powerful low-frequency interference significantly reduces the dynamic range of the hydroacoustic receiver and can cause serious

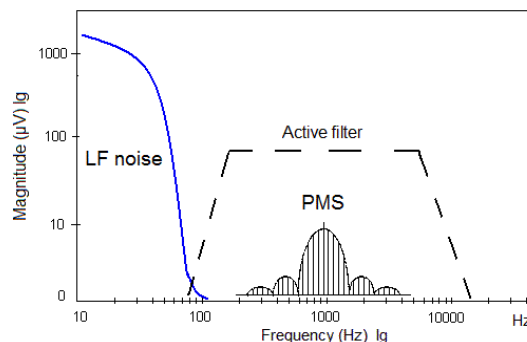


Figure 1. Input signals of a hydroacoustic receiving station.

disruption of the ADC operation when digitizing the analog signal. Therefore, it is highly desirable to construct AAF to suppress this low-frequency noise, i.e., to design AAF as a bandpass filter suppressing all signals above the Nyquist frequency and the noise of the low-frequency interference. Thus, the main requirements for an anti-aliasing filter are:

- efficient signal suppression above the Nyquist frequency;
- efficient low-frequency noise suppression;
- minimal amplitude and phase distortion in the filter pass-band;
- steep cutoff and minimal width of the transition band;
- wide dynamic range of the AAF;
- discrete values of filter components (resistors and capacitors).
- minimal dimensions and weight.

To process hydroacoustic signals in a low and ultralow frequency range these requirements, a high-order active RC filter should be used. The main

advantages of active filters are their small size, low power consumption, ease of adjustment, sufficient stability with not too stringent selectivity requirements, as well as a low cutoff frequency, which is practically impossible to achieve with passive RLC filters. The active filters, as devices for frequency selection of the input signal, are usually designed based on the requirements for their frequency responses, such as magnitude response (MFR), phase response (PFR), group delay time (GDT), and frequency dispersion of the signal. Developing an active filter synthesis method which would meet such requirements is a topical problem.

At present, the analytical calculation method is predominant among the active filter design options. This method consists of the following steps:

- approximation step that involves designing the transfer function to provide the required frequency response of the filter;
- implementation step that involves selecting the filter circuit according to the poles and zeros of the filter transfer function and calculating its elements.

This approach is used, for example, in the well-known online calculators such as Analog Filter Wizard by the Analog Device company [2] and online-program FilterPro version 3.1 by the Texas Instruments [3].

Disadvantages of the active filter analytical design are:

- fundamental impossibility of a multifunctional active filter synthesis, since there are only requirements for the magnitude response taken into account and no other can be fulfilled;
- practical impossibility of synthesizing an active filter with an arbitrary form of the frequency response, since approximation of an arbitrary response function is a separate and complicated problem;
- very limited active filter topology selection. In the FilterPro calculator a high-order cascaded filter can only be implemented on the Rauch sections with a Multiple-Feedback or on the Sallen-Key sections with a voltage-controlled voltage source and the same topology must be used for any filter section;
- no possibility to design the solution in a discrete space. The filter components can only take the values determined by standard series from E6 (with a 20% error) to E192 (a 0.5% error), for the electronic elements manufactured by industry;
- for analytical calculation of the active filter response most commercial programs use models of ideal operational amplifiers;
- no other external conditions and functional limitations (e.g. signal scaling conditions in cascaded filters) can be directly taken into account when designing the active filter.

The active filter search design in a discrete parameter space (resistor and capacitor values) is an alternative approach, which largely does not have the above disadvantages. The design task is reduced to the problem of non-linear mathematical programming [4,

5], the general idea of which is to link the technical solution to a clear invariant mathematical feature – the extremum of the active filter aggregate quality function (objective function)  $F(X)$ , where  $X$  is the vector of the searched filter component values. This function for any design task can always be formed on the basis of the required filter characteristics (in computer packages this is usually done with a graphics user interface). Having this function, we can reduce the synthesis problem to a minimizing  $F(X)$  procedure to find the coordinates of the global extremum (optimal active filter component values  $X^0$ ) in a continuous or discrete design space. Minimization is made by search methods [6].

In this paper the readers are introduced to a search solution of the synthesis problem obtained by using the BARC search program, developed at a radio engineering department by the authors. It is intended for a multifunctional search design of active devices with an arbitrary structure in a continuous or discrete parameter space.

The functional requirements met by using this program on an example of a discrete synthesis of a band-pass AAF for a hydroacoustic receiving station are:

1. filter bandwidth is 180 – 800 Hz;
2. transfer ratio is  $K_u = 1.0$  (0 dB);
3. unevenness of  $K_u$  in the passband is 0.5 dB;
4. suppression factor in the frequency range from 50 to 3000 Hz is not less than 40 dB;
5. phase distortion in the passband is less than  $60^\circ$ ;
6. filter components are of the E12 series (10%);
7. minimum dimensions and weight.

Fig. 2 shows the required AAF magnitude frequency response.

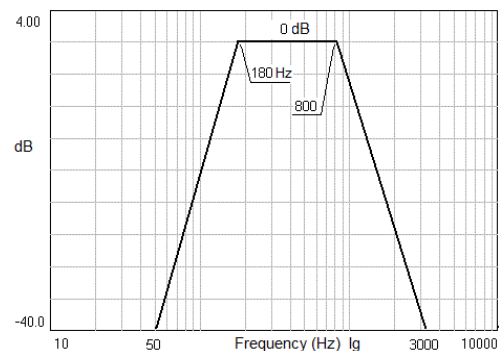


Figure 2. Required AAF magnitude frequency response.

## 2 MACROMODELLING OF THE OPERATIONAL AMPLIFIER

At the first stage of the amplifier design an operational amplifier (op amp) is selected on the basis of which the AAF is realized, and form its linear substitution macromodel formed.

The methodology for constructing macromodels of real integrated circuits and operational amplifiers is well developed [7, 8]. To implement the above technical specification, the MAX44241 operational amplifier of

the Maxim Integrated company is quite suitable since its frequency range meets the requirements with a large margin and the transfer function is quite simple and has only one salient inflection point [9]. Such dependence can be modeled with a high accuracy by setting only one pole of the macromodel transfer function (Fig. 3). A single-pole model requires minimal resources of the computer main memory when modelling the op amp since it has a simple topology and only one internal node.

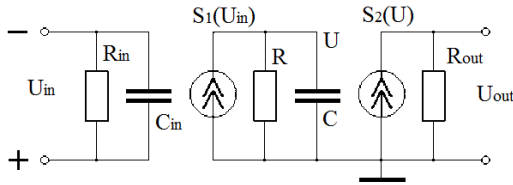


Figure 3. Single-pole macromodel of the MAX44241 opamp

The linear substitution model has the same input and output impedances and the same transfer function (Fig. 4), as a real op amp. The inflection point at a frequency of 1 Hz corresponds to a negative real pole and is modelled by  $U_{in}$  voltage-controlled current source S1 with a conversion ratio 1 a/v, resistor  $R = 300 \text{ k}\Omega$  and capacitor  $C = 1 \text{ }\mu\text{F}$ . The values of other parameters of the macromodel are  $R_{in} = 1 \text{ G}\Omega$ ,  $S_{in} = 2 \text{ pF}$ ,  $R_{out} = 300 \text{ }\Omega$ , conversion ratio 1.6 a/v for current source S2 controlled by voltage  $U$ . As can be seen, some of the parameters of this macromodel are taken from the op amp passport data ( $R_{in}$ ,  $R_{out}$  ones) [9], and the other are optimized with the BARC package for the required frequency response of the MAX44241 op amp.

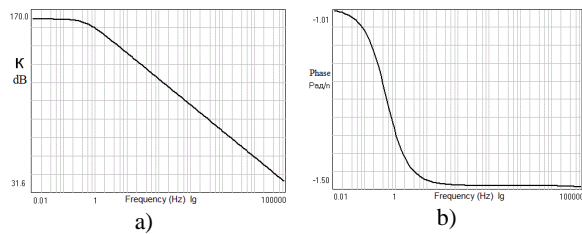


Figure 4. Magnitude (a) and phase (b) frequency response of the MAX44241 op-amp macromodel

### 3 THE FILTER TOPOLOGY

At the second stage of the amplifier design, the active filter topology is selected. As known from the experience in practical developing, filters with a simple cascade topology on the sections of the same structure hardly fulfill complex functional requirements. Many of the disadvantages of this topology can be eliminated by applying cascaded filters consisting of the first- or second-order sections of different structures connected by a negative feedback [10-11]. The sensitivity to changes in the parameters in such filters is much smaller, and the possibility of realizing the required characteristics is significantly increased. In addition, by

cascading the coupled blocks in such filters it is possible to reduce the undesirable propagation of high frequency signals to the filter output occurring in most active filters with a multi-loop feedback.

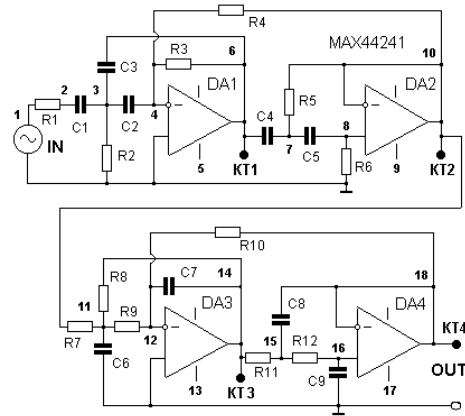


Figure 5. Polynomial active bandpass filter.

Let us choose an eight-order polynomial AAF with connected cascades of two HPH and two LPF sections using an interstage negative feedback. Using sections of different frequencies enables independent control of the slope of the low- and high-frequency cut-offs just by adding additional sections to the low- or high-pass cascade of the designed filter.

The transfer function of the polynomial filters is

$$K(p) = \frac{K_0}{\nu(p)}, \quad (1)$$

where  $\nu(p)$  is the Hurwitz polynomial of  $n$ th degree  $n$  with all its roots lying in the left half-plane of the complex frequency  $p = \sigma + j\omega$ , and constant factor  $K_0$  determines the transfer ratio of the filter at a zero frequency. Thus, a polynomial filter has no zeros of the transfer function and all its poles are finite. This ensures a higher phase linearity and stability of the polynomial linear dynamic system, as well as its simpler implementation compared to linear circuits with finite zeros of the transfer function.

Fig. 5 shows a filter with such a topology for the alternating current component. When analyzing in a frequency domain, the selected scheme of the filter is treated as a linear stationary circuit with a replacement of a real operational amplifier with its linear macromodel and numerical computation of the complex transfer function in a given frequency range by the node voltage method (the numbers in the diagram denote the numbers of independent nodes). This makes it possible to model and synthesize active filters of diverse topologies. To enter the selected topology into a synthesis program, we use the built-in topological interface to create a source data file for solving a specific synthesis problem, indicating the number of the variable parameters, their initial values and the limits of the parameters change, as well as possible discretization and duplication of the parameters, if necessary.

### 4 DISCRETE SYNTHESIS OF THE FILTER

When synthesizing an active band-pass filter, a user enters the required magnitude frequency response of the filter into the BARC program in graphic mode (Fig. 2). The program forms the objective function according to the criterion of the minimum root-mean-square deviation

$$F(\mathbf{X}) = \sqrt{\frac{1}{P} \cdot \sum_{n=1}^P [Y_n(\mathbf{X}) - Y_n^T]^2}, \quad (2)$$

where  $Y_n(\mathbf{X})$  is the current value of the filter frequency response at the  $n$ -th discrete frequency, and  $Y_n^T$  is the required response value at this discrete frequency.

The polynomial band-pass active filter synthesis problem can be written by the objective function:

$$F^o(\mathbf{X}^o) = \min F(\mathbf{X}) \quad \mathbf{X} \in DX \quad (3)$$

$$DX : \begin{cases} R_i \in E12 \\ C_i \in E12 \\ 100\Omega \leq R_i \leq 1M\Omega \\ 100\text{ pF} \leq C_i \leq 4,42\text{ nF} \end{cases}, (4)$$

$$0,6 \leq |K_i(j\omega)| \leq 1,2 \quad i = \overline{1,3} \quad (5)$$

The solution to the discrete programming extremum problem is sought on a discrete set. All the variables are restricted to belong to multidimensional design space  $DX$  which consists of resistor and capacitance values from the E12 series. This series permissible deviation from the nominal value is less than 10%. Thus the procedure of rounding the filter component values, which is necessary after getting the solution values, for example, by FilterPro 3.1, is substituted by a set of discrete multidimensional design space before active filter synthesis, and the error of the filter practical realization is zero. Constraints (4) determine the range of these discrete variable component values. In the problem under consideration, all 20 external component values of AAF were placed under optimization, except the values of the op amp macromodels and the input signal generator resistance  $R1 = 600 \Omega$ . Functional constraints (5) scaled the cascade gain factors (by the KT1-KT3 reference points) to the set interval, ensuring stability of the designed filter in a wide dynamic range of the input signals. Signal scaling (4) was carried out using windows of a graphic user interface, where for the filter sections we set the upper and lower limits of the maximum gain in a given frequency range.

To solve the extremum problem (3) numerically, the multidimensional discrete optimization BARC software package is employed. BARC uses an effective global minimization algorithm on Gray's code grid [12], adopted to the solution search when the design space is restricted by one of the standard series from E6 to E192. The software package carries out a direct search of the global minimum of the multidimensional objective

function (2) beginning from the starting point, which is set by a user (initial guess), and making sequential iterative steps to the global minimum point  $F0(\mathbf{X}0)$ . This ensures fulfillment of the active filter required characteristics.

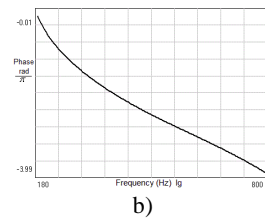
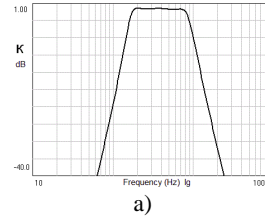


Figure 6. AAF computer calculated frequency responses

- a) Magnitude response
- b) Phase response

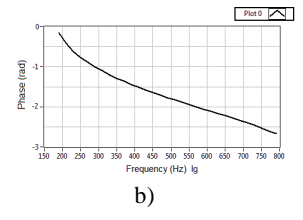
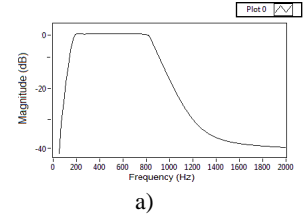


Figure 7. AAF measured frequency responses

- a) Magnitude response
- b) Phase response

It takes less than 10 minutes to solve this task on a standard personal computer. Table 1 lists the optimal component values, and Table 2 shows the synthesized AAF characteristics obtained by computer calculation (6) and experimental measurements (7). Figs. 6 and 7 demonstrate the frequency responses. To find the filter frequency responses, a real signal is measured with an automatic panoramic measuring system developed in the visual system-design platform LabVIEW.

Table 1. Filter component values.

Resistors				Capacitors			
R1	600 ohm	R7	12 K	C1	3.9 nF	C7	2.7 nF
R2	39 K	R8	82 K	C2	15 nF	C8	2.2 nF
R3	100 K	R9	27 K	C3	5.6 nF	C9	100 nF
R4	68 K	R10	820 K	C4	2.2 nF		
R5	47 K	R11	180 K	C5	3.3 nF		
R6	820 K	R12	820 K	C6	47 nF		

Table 2. AAF characteristics.

Filter characteristics	Computer synthesis	Measurement
1. Pass band, Hz	180 – 800	178 – 810
2. Attenuation at frequencies of from 50 to 3000 Hz is no less than, dB	- 40	- 42
3. Passband gain , dB	0	0.072
4. Gain unevenness, dB	0.42	0.65
5. Phase distortions in the passband	54	47

The tables and graphs demonstrate that all the requirements for the bandpass polynomial AAF are fulfilled with a high accuracy. The high level of signal scaling is achieved as a result of the synthesis. The maximum values of the gain factor of the sections (for

the reference points KT1, KT2 and KT3) are within the limits of the given interval  $\{0.6 - 1.2\}$ .

We investigate the behaviour of the objective function at the optimum point by analyzing its slices. Slices along variables R9 and C7 (Fig. 8) demonstrate that the objective functions in active filter synthesis problems are complex and multimodal. Discrete minimization of such functions is a very difficult task. Nevertheless, the algorithmic search package BARC successfully copes with this task, showing a high reliability and efficiency.

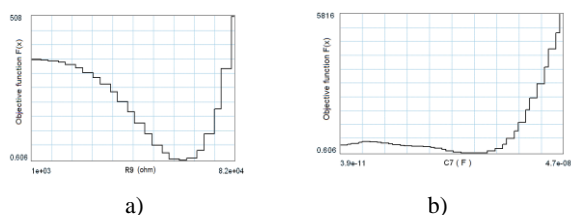


Figure 8. Slices of the objective function along variable R9 (a) and variable C7 (b).

## 5 CONCLUSION

The requirements for the AAF frequency selective properties in hydroacoustic receiving stations are high. That is why it is necessary to improve the methodology for filter modelling and synthesis. Search methods of discrete optimization in an active filters design are a promising and modern alternative to their traditional analytical calculations. The main difference is that when synthesizing AAF with the required characteristics, a direct search of the active filter component values in a multidimensional discrete space is carried out at the design stage. The criterion for searching an effective solution is that the filter aggregate current functioning corresponds to its required functioning. Using modern algorithmic discrete minimization systems solves this problem reliably and effectively with fulfilling all external requirements and limitations to the filter operation. This makes it possible to significantly improve the active filter quality and to shorten the time of the filter development.

The materials given in this paper demonstrate that the search synthesis of the active filters by discrete nonlinear programming makes the following possible:

- to synthesize an active filter with the required frequency responses with any topology and to numerically compute its complex transfer function by the node voltage method;
- to synthesize the filter according to a set of the required characteristics and to control easily the priority of the functional characteristics during the filter synthesis;
- to take into account the features of a real operational amplifier by its macromodelling in a frequency domain when designing a filter;

- to ensure a high reliability of the solution to the extreme filter synthesis problem found by the global search minimization algorithm;
- to scale amplification of the signal in the filter sections directly while carrying out the search synthesis. So there is no need to use indirect methods of amplification scaling;
- to find the filter component values belonging to one of E6–E192 standard series, according to which electronic components are manufactured by industry, therefore realization of the design solutions does not cause any practical difficulties.

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