

# A simplified Procedure to determine the Earth-fault Currents in Compensated Networks

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**Abstract.** The paper proposes a simplified approach for determination of the earth-fault currents to be used in earth-fault compensated systems. By estimating the relative magnitudes of the equipment parameters, the equivalent circuit in symmetrical components at a single-pole fault is significantly reduced enabling the electrical parameters of the impedances to be directly used.

**Keywords:** earth-fault current, earth-fault calculation, compensated power networks

## Učinkovit izračun tokov zemeljskega stika v omrežjih z resonančno ozemljeno nevtralno točko

Članek predstavlja poenostavljen pristop za določanje toka zemeljskega stika, ki je še posebej primeren za omrežja z resonančno ozemljeno nevtralno točko. Zaradi ocenjenih relativnih vrednosti parametrov posameznih elementov omrežja se v primeru enopolnih okvar nadomestno vezje v sistemu simetričnih komponent močno poenostavi. Tak pristop omogoča direktno uporabo znanih parametrov impedanc posameznih elementov omrežja.

## 1 INTRODUCTION

The phase-to-earth-faults, i.e. in the medium-voltage networks constitute the major part of the fault events (see figure 1). Especially in view of the so-called extinction limit and also, if dangerous touch voltages occur at such faults, the determination of the fault current is very important.

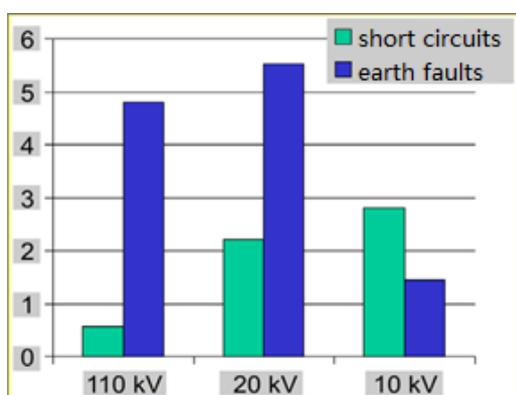


Fig. 1. Statistics for the single- and multi-phase faults, over voltage levels [1]

Recently, the question of usefulness of operating the earth-fault-compensated networks has been discussed<sup>1</sup>. As a result of the current state of discussion and regardless of the justified exceptions, such as aged cable networks, this operating regime is often useful. Especially in mixed networks, i.e. those with a large proportion of the overhead lines, operating with no fault-compensated neutral point would lead to a massive increase in the earth-fault current which would affect the substation earthing design and might necessitate upgrading the plant earthing apparatus.

According to the valid standards the plant safety at an earth-fault, the earth-fault current needs to be calculated.

<sup>1</sup> In the case of resonant earthed networks (with Petersen coil) the capacitive earth-fault current is compensated in such a way that at the fault point the earth-fault current as well as the resulting earth potential rise (EPR) are minimized by the arc suppression coil (Petersen coil). In most cases, this induces an automatic termination of the fault arc in overhead systems. Thus the necessity of switching off the faulted line is eliminated. The resonant earthed network can be operated continuously in the event of a single-pole fault and the reliability of supply of this network is not impaired.



Fig. 2. Standing arc in a substation switchgear

## 2 TASK

The earth-fault current is calculated mainly for two reasons:

- To determine of the residual current at the earth-fault location (i.e. the earth potential rise (EPR) and the associated hazards)
- To calculate the phasors of the zero-sequence currents and the accompanying residual voltages at various points in an electrical network in order to correctly determine the protection settings concerning their sensitivity and pick-up reliability correctly. This approach is often referred to as “earth-fault engineering.”

Usually, the method of symmetrical components is used to accomplish the above. In practice, the use of the network calculation programs for a detailed analysis is recommended.

Experience shows that using the method of symmetrical components requires a considerable amount of theoretical knowledge. Because of the necessary matrix operations and scaling factors, it is often complicated and may lead in many cases to calculation errors.

## 3 METHODOLOGY

Based on the theory of symmetrical components, the equivalent circuit for a phase-to-earth-fault is a series circuit of positive-, negative-, and zero-sequence components (see Fig. 3). It is simplified by neglecting the positive- and the negative-sequence system in order to calculate the earth-fault current (ground fault in the earth-fault-compensated network).

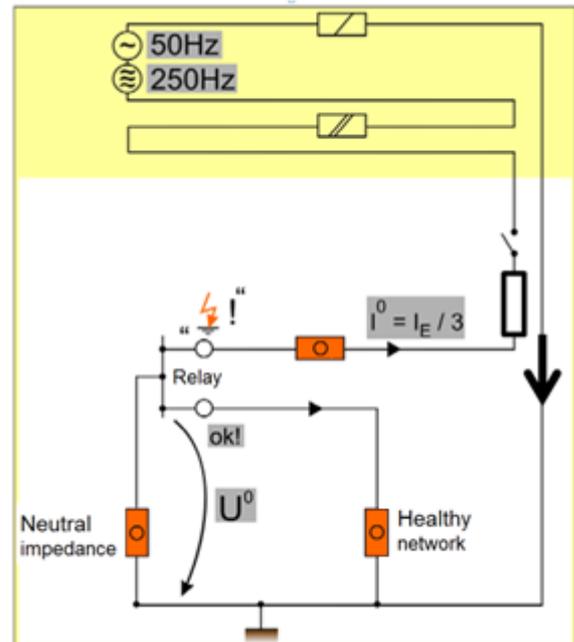


Fig. 3. Equivalent circuit for the single-pole fault with relevant parameters, [1]

The derivations are based on the theory of symmetrical components. Accordingly, the methodology is explained stepwise.

The accuracy of the simplified calculations is usually  $\pm 10\%$ . The zero-sequence impedances of any neutral star-point transformer and its positive-sequence impedances or of the upstream high-voltage grid - expressed by the short-circuit power  $S_k$  -, as well as the angle of the earth return-path factor (“ $k_0$ ”), are neglected.

**Comments regarding the simplifications:** Numerical analysis of the current flow - represented by the symmetrical component system - shows that the series connection of the positive-, negative-, and zero-sequence components can be interpreted as a series connection of the phase voltage of phase L1 as a driving source voltage with an internal source impedance, with a high-ohmic load. The positive- and negative-sequence system can be considered as a voltage source

impedance, whereas the zero-sequence system represents the source load.

As to the typical numerical values, the series connection of the positive- and negative-sequence is only a part of the total impedance (positive-, negative- and zero-sequence) and can therefore be neglected.

**Comment on the following derivation steps:** Starting from a preliminary stage (considering the earth-fault current in a TN-system), the earth-fault current is determined consecutively for the following network types:

- earth-fault in a TN-system (preliminary stage)
- network with a current-limiting star-point resistor
- network with an insulated neutral
- network with a Peterson arc-suppression coil: fundamental and harmonic currents
- network with a Peterson arc-suppression coil and short-term resistance earthing of the neutral point

At selected locations, the results are demonstrated by simple calculations. In each of the above steps, only the network elements are added which are necessary to understand the methodology and to obtain the required calculation accuracy.

#### 4 EARTH-FAULT CALCULATION IN A TN SYSTEM (PRELIMINARY STAGE)

In a first step, an earth-fault in a TN system (= short circuit) between a conductor and PEN conductor is considered (see Fig. 4 and 5)

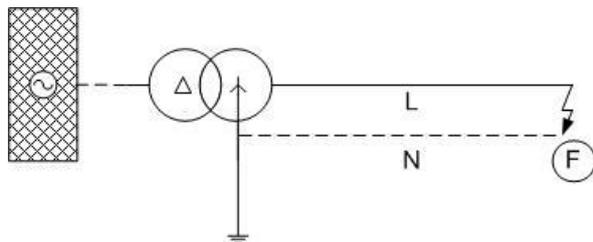


Fig. 4: earth-fault in a TN system: circuit diagram; F ... fault location

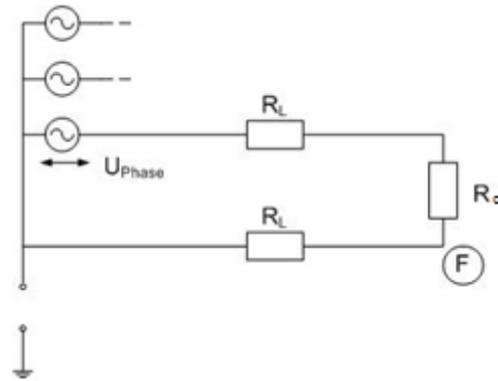


Fig. 5: Physical current flow during earth-fault in a TN system; F ... fault location

According to the illustration the earth-fault current is given by

$$I_{F, TN} = U_{Ph} / (R_L + R_c + R_L) = U_{Ph} / (2 \cdot R_L + R_c) \quad (1)$$

$I_{F, TN}$  ... earth-fault current at the fault location

$U_{Ph}$  ... phase-to-earth voltage

$R_L$  ... resistance of the phase conductor

$R_c$  ... contact resistance at the fault location

When taking into account the influence of the inductive line impedance, this becomes

$$I_F = U_{Ph} / (Z_L + R_c + Z_{PEN}) \approx U_{Ph} / (2 \cdot Z_L + R_c) \quad (2)$$

$Z_L$  ... impedance of the phase conductor

$Z_{PEN}$  ... impedance of the PEN conductor

This relationship can be simplified in approximation to the system of symmetrical components, neglecting the series connection of the positive and negative sequence respectively (see figure 6).

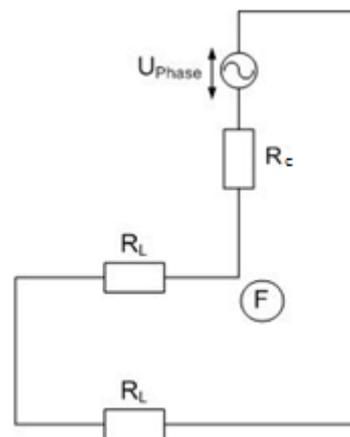


Fig. 6. Simplified representation of a ground fault in TN systems - equivalent circuit, F ... fault location

### 5 CALCULATION OF THE EARTH-FAULT CURRENT IN A RESISTANCE-GROUNDED NETWORK

To answer the core question of the earth-fault calculation "How is the current path of the earth-fault current closed, looking at the return-path of the current loop from the ground into the elsewhere highly galvanically insulated power network?" it is necessary to consider the following current loop (see figure 7):

- substation busbar → phase conductor  $Z_L$  → line-to-earth transition resistance  $R_c$  → earth-return-path impedance  $Z_m$  → star-point resistance  $R_{Stp}$  → substation busbar

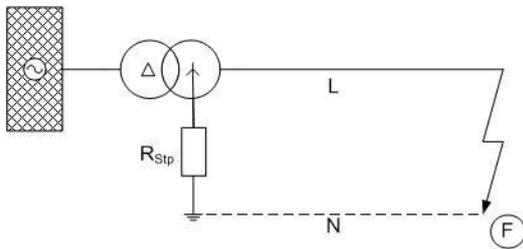


Fig. 7. Earth-fault in a resistance-grounded network - Electrical diagram; F ... fault location

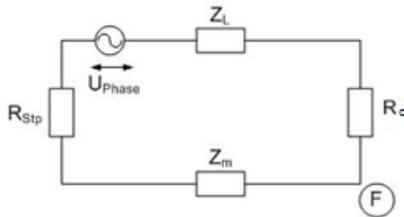


Fig. 8. Physical current flow at an earth-fault in a resistance-grounded network; F ... fault location

For the relationship between line impedance  $Z_L$  and earth return circuit impedance  $Z_m$ , one can use the well-known expression of from the power-system protection.

$$Z_m = k_0 \cdot Z_L \text{ with } k_{0,cable} = 0,8 \text{ or } k_{0,OHL} = 1$$

In addition, with a good approximation the circuit impedance  $Z_L$  can be replaced by the line reactance  $X_L$ . As shown in figure 9, the earth-fault current  $I_F$  is given by (1) where  $U_N$  is the nominal voltage,  $R_c$  is the contact resistance at the fault location and  $R_{Stp}$  is the neutral-point resistance.

$$I_F = \frac{\frac{U_N}{\sqrt{3}}}{\sqrt{(R_c + R_{Stp})^2 + [(1 + k_0) \cdot X_L]^2}} \quad (1)$$

As shown above, this value results also (in accordance with the system of symmetrical components) from the slightly modified representation shown in Fig. 9.

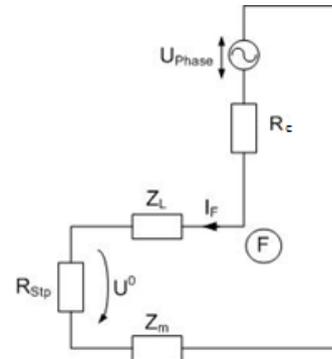


Fig. 9. Earth faults in a resistance grounded network - equivalent circuit; F ... fault location

In practice the fault current is dominated by the neutral point resistance  $R_{Stp}$ .

### 6 CALCULATION OF THE EARTH-FAULT CURRENT IN A NETWORK WITH AN INSULATED NEUTRAL

To answer above question for this case the earth-to-conductor capacitances have to be considered (see Fig. 10).

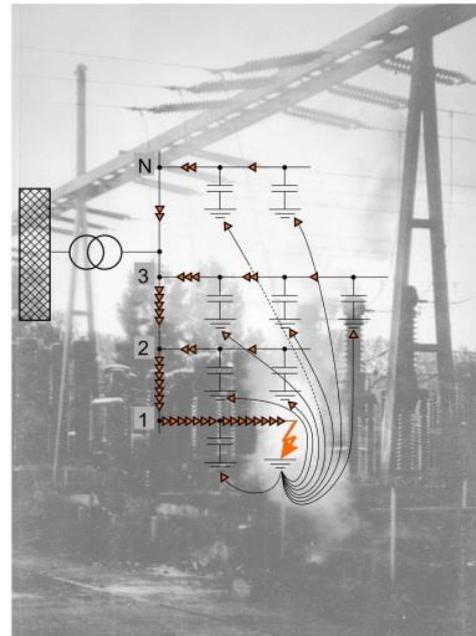


Fig. 10. Arc and capacitive earth-to-conductor current paths

In a network with a galvanically insulated neutral, the path of the earth-fault current returning from the fault location is divided into different sub-current paths ( $I_{F,1}$  to  $I_{F,n}$ ) by their respective single earth-to-conductor capacitances (see Fig. 11).

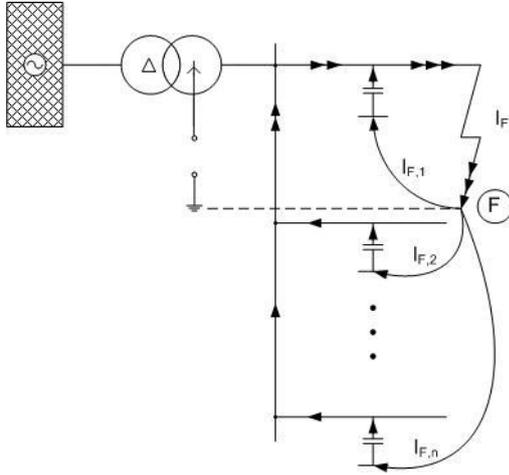


Fig. 11. Physical current flow in case of an earth fault in a network with an insulated neutral; F ... fault location

By adding these individual earth-to-conductor capacitances to the total capacity  $X_{cap}$  as well as the individual sub-current paths to the total current ( $I_{F,CAP}$ ), the following schematic diagram is obtained (Fig. 12).

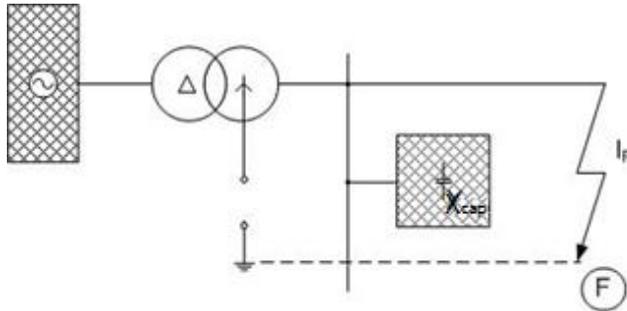


Fig. 12. Earth fault in a network with an insulated neutral - electrical diagram; F ... fault location

As shown in Figs. 11 and 12, the earth-fault current  $I_F$  consists of the capacitive current contributions of each network segment as follows:

In a cable network with a cable length  $l_{cable}$ , each km of the cable length contributes an earth fault current of about 3 A. 1 km of the overhead line contributes approximately 1/30 of this value. As a result, the capacitive earth-fault current flowing in the network is

(with a neglectable line-to-ground transition resistance, worst-case):

$$I_F = l_{cable} \cdot i'_{cable} + l_{OHL} \cdot i'_{OHL} \quad (2)$$

where:  $l_{cable}, l_{OHL}$  - total length of the cables/overhead lines,  
 $i'_{cable}$  - capacitive current contribution per kilometer of the cable  
 $i'_{OHL}$  - capacitive current contribution per kilometer of the overhead line

The value calculated according to (2) can also be derived (similarly to the system of symmetrical components) from a slightly modified presentation shown in Fig. 13.

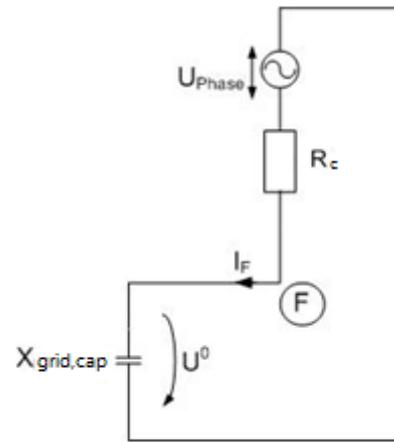


Fig. 13. Earth fault in a network with an insulated neutral - equivalent circuit; F... fault location

According to Fig. 13, the equivalent total network capacitance  $X_{grid,cap}$  for the fault location "F" is the internal impedance of the network.

$$X_{grid,cap} = \frac{U_{Phase}}{l_{cable} \cdot i'_{cable} + l_{OHL} \cdot i'_{OHL}} \quad (3)$$

**Remark:** If the line-to-ground transition resistance cannot be neglected ( $R_c > \text{some } 100 \Omega$ ), the earth-fault current is calculated from the complete current loop:

- Transformer  $\rightarrow$  line-to-ground transition resistance  $R_c$
- $\rightarrow$  equivalent total network capacitance  $X_{grid,cap}$

$$I_F = \frac{\frac{U_N}{\sqrt{3}}}{\sqrt{R_c^2 + X_{grid,cap}^2}} \quad (4)$$

### 7 CALCULATION OF THE EARTH-FAULT CURRENT IN A RESONANCE-GROUNDED NETWORK – CALCULATION OF THE 50 HZ COMPONENTS

To answer the above core question for this case it is necessary that besides the earth-to-conductor capacitances the newly created current return path through the Petersen arc suppression coil is also considered (see Fig. 14).

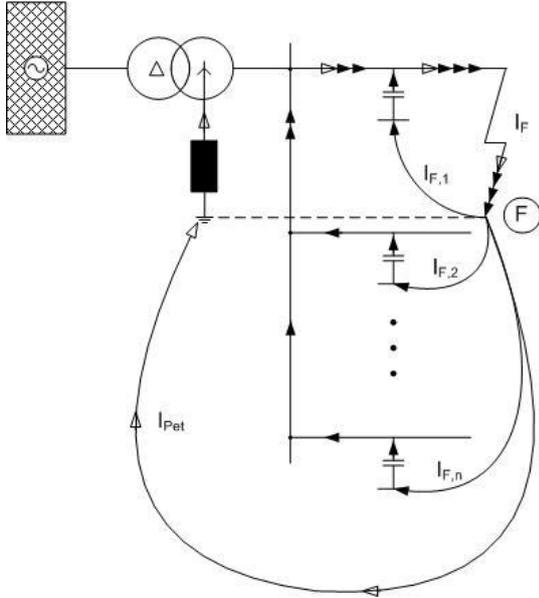


Fig. 14. Physical current flow for an earth fault in an earth-fault compensated network; F ... fault location

As in the procedure for a network with an insulated neutral, the following circuit diagram (Fig. 15) is used:

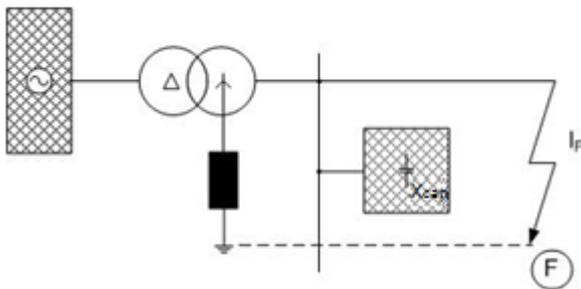


Fig. 15. Earth fault in an earth-fault compensated network; F ... fault location

As shown in Figs. 14 and 15, the earth-fault current  $I_F$  is again obtained by superposing the capacitive current contribution of individual sections of the network ( $I_{grid, cap, ef}$ ), but this time it is also superimposed with the inductive coil current  $I_{Pet}$  of an opposite sign. As a

result, the following total earth-fault current is obtained (with the line to ground transition resistance at the fault location):

$$I_F = I_{Pet} - I_{grid, cap, ef} \tag{5}$$

$$I_F = \frac{U_{Phase}}{X_{Pet}} - (I_{cable} \cdot i'_{cable} + I_{OHL} \cdot i'_{OHL}) \tag{6}$$

$$I_F = m \cdot I_{grid, cap, ef} \tag{7}$$

$I_{Pet}$  ... current through the Petersen arc-suppression coil  
 $X_{Pet}$  ... reactance of the Petersen arc-suppression coil  
 $m$  ... degree of over-compensation of the network (in p.u.)

As in the previous section the above value (7) can be obtained from a slightly modified presentation (similarly as for the system of symmetrical components) given in Fig. 16.

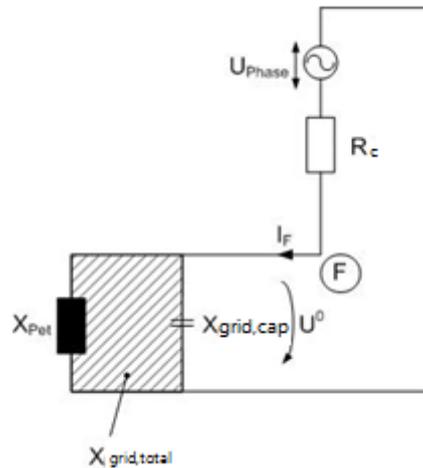


Fig. 16. Earth fault in an earth-fault compensated network; equivalent circuit

The total reactance of the network given in Fig. 16 can be calculated using (8).

$$X_{grid, total} = \frac{U_{Phase}}{I_F} = \frac{U_{Phase}}{m \cdot I_{grid, cap, ef}} \tag{8}$$

**Remark:** The value of  $X_{grid, total}$  can also be obtained for control purposes from the following relationship

$$X_{grid, total} = X_{grid, cap} // X_{Pet} = (-U_{Phase}/I_{grid, cap, ef}) // (U_{Phase}/I_{Pet})$$

The earth-fault current is calculated from the complete current loop according to (4).

- substation busbar → line-to-ground transition resistance  $R_c$  → source reactance of the network  $X_{grid,total}$  → substation busbar

### Numerical example 1:

- The earth fault in an extended 20 kV cable network (nine feeders of 10 km each)
- The Petersen arc suppression coil is by 3.7 % overcompensated
- The line-to-ground transition resistance is 100  $\Omega$

$$U_{Phase} = 20.000 \text{ V} / \sqrt{3} = 11.547 \text{ V}$$

$$\begin{aligned} I_{grid, cap, ef} &= I_{cable} \cdot i'_{cable} + I_{OHL} \cdot i'_{OHL} \\ &= \text{nine feeders of } 10 \text{ km} \cdot 3 \text{ A/km} + 0 \\ &= 270 \text{ A} \end{aligned}$$

$$\begin{aligned} I_{Pet} &= (1+m) I_{grid, cap, ef} = (1 + 0,037) \cdot 270 \text{ A} \\ &= 280 \text{ A} \end{aligned}$$

$$\begin{aligned} X_{grid, total} &= U_{Phase} / I_F \\ &= U_{Phase} / (m \cdot I_{grid, cap, ef}) \\ &= 11.547 \text{ V} / 10 \text{ A} \\ &= 1155 \Omega \end{aligned}$$

$$\begin{aligned} I_F &= U_{Phase} / \sqrt{(R_c^2 + X_{grid, total}^2)} = 11.547 / \sqrt{(100^2 + 1155^2)} \\ &= 9,9 \text{ A} \end{aligned}$$

## 8 CALCULATION OF THE EARTH-FAULT CURRENT IN A RESONANCE- GROUNDED NETWORK – CALCULATION OF THE 250-HZ-COMPONENTS

When calculating the 250 Hz conditions, the following is taken into account (see Fig. 17):

1. The driving voltage is the 250 Hz component of the pre-fault phase-to-earth voltage  $U_{Phase, 250} = p_{250} \cdot U_N / \sqrt{3}$ , where  $p_{250}$  is the level of the 5th harmonic in the line voltage.
2. At 250 Hz each capacitive reactance is five times smaller than at 50 Hz.
3. At 250 Hz the reactance of the Petersen coil is five times higher than at 50 Hz. Therefore, its reactance in relation to the system capacitances can be neglected due to the parallel circuit
4. The line reactances are five times higher than at 50 Hz and should not be neglected.

In accordance with Fig. 17, the Petersen coil  $X_{Pet}$  is omitted because of the now negligibly high impedance. This means that at higher frequencies, an earth-fault-compensated network behaves like a

network with an insulated neutral. The factor of the line reactances taken into account is  $f/50\text{Hz}$ .

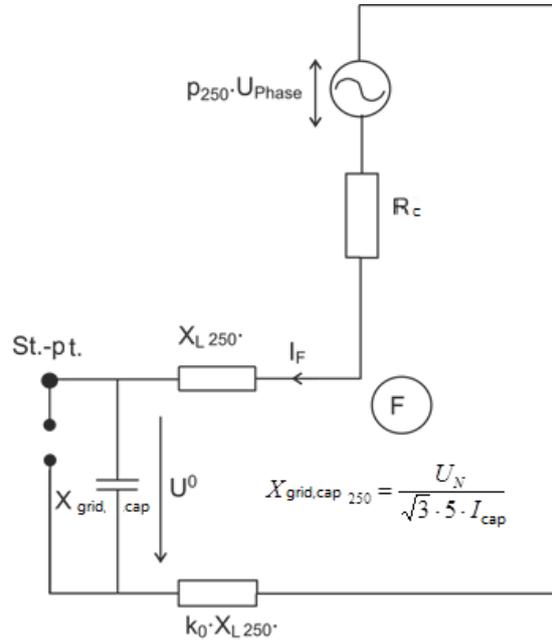


Fig. 17. Earth fault in an earth-fault compensated network; equivalent circuit

**Remark 1:** To allow for simplification (from the whole line impedance only the line reactance is taken) at a certain line length, the reactive components are mutually canceled which leads to a resonant condition. In such an idealised case, the current is infinite which is not realistic. As the effective resistances are still present, the harmonic content in the earth-fault current is effectively attenuated. It is only here that an exact calculation is required.

**Remark 2:** The value of  $X_{grid, total}$  is obtained for control purposes from the following relationship:

$$X_{grid, total, 250} = (X_{grid, cap, 50} / 5) // (X_{Pet} \cdot 5) \approx (-U_{Phase} / I_{grid, cap, ef}) / 5$$

**Remark 3:** At 250 Hz the reactance value of the loop from the fault location to the substation busbar and back is five times the value of the 50-Hz reactance.

The earth-fault current is calculated from the complete current loop:

- substation busbar → line-to-ground transition resistance  $R_c$  → phase conductor  $X_{L, 250}$  → network capacity  $X_{grid, total, 250}$  → earth return path impedance  $Z_{m, 250} = k_0 \cdot X_{L, 250}$  → substation busbar

$$I_{F, 250} = p_{250} \cdot \frac{\frac{U_N}{\sqrt{3}}}{\sqrt{R_c^2 + [(1+k_0) \cdot X_{L, 250} - X_{grid, cap, 250}]^2}} \quad (9)$$

**Numerical example 2:**

- The earth fault in an extended 20 kV cable network (nine feeders of 10 km each with  $x' = 0,13 \Omega/\text{km}$ )
- The Petersen arc-suppression coil is by 3.7 % overcompensated
- The line-to-ground transition resistance is 100  $\Omega$
- The fault location is 3 km away from the substation busbar

$$X_L = 3 \text{ km} \cdot 0.13 \Omega/\text{km} = 0.39 \text{ Ohm}$$

$$k_0 = 0.7$$

$$p_{250} = 2 \%$$

$$I_{\text{grid, cap, ef}} = I_{\text{cable}} i'_{\text{cable}} + I_{\text{OHL}} i'_{\text{OHL}} \\ = \text{nine feeders} \cdot 10 \text{ km} \cdot 3 \text{ A/km} + 0 = 270 \text{ A}$$

$$U_{\text{Phase, 250}} = p_{250} \cdot U_N / \sqrt{3} = 0,02 \cdot 20.000 \text{ V} / \sqrt{3} = 231 \text{ V}$$

$$X_{\text{grid, total, 250}} \approx (-U_{\text{Phase}} / I_{\text{grid, total}}) / 5 \\ = (11547 \text{ V} / 270 \text{ A}) / 5 \\ = 8.6 \Omega$$

$$X_{L, 250} = x' \cdot l \cdot 5 = 3 \text{ km} \cdot 0.13 \Omega \cdot 5 = 1.95 \Omega$$

$$I_{F, 250} = p_{250} \cdot (U_N / \sqrt{3}) / \sqrt{(R_C^2 + [(1+k_0) \cdot X_{L, 250} - X_{\text{grid, cap, 250}}]^2)} \\ = 231 / \sqrt{(100^2 + [(1+0.7) \cdot 1.95 - 8.6]^2)} = 2.3 \text{ A}$$

**9 CALCULATION OF THE EARTH-FAULT CURRENT IN A RESONANCE- AND SHORT-TIME RESISTANCE-GROUNDED NETWORK**

In a resonance-grounded network, a more or less low-ohmic resistance is in some cases briefly switched parallel to the Petersen coil to improve the earth-fault location by evaluating the resistive component. A resistor parallel to the Petersen coil is taken into account. Hence, another current path is activated through this neutral-point resistor causing resistive component  $I_R$ . Figs. 19 and 20 show the physical current flow and the derived circuit diagram, respectively.

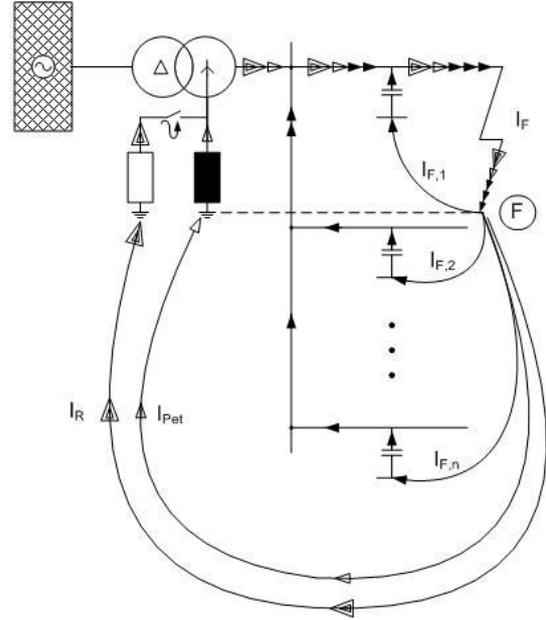


Fig. 18. Physical earth-fault current flow in a network with a resonance- and short-term resistance-grounded neutral point; F ... fault location

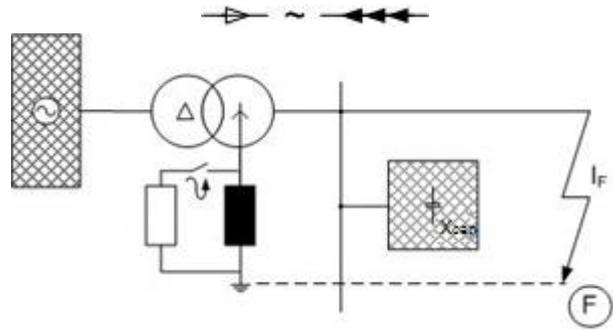


Fig. 19. Earth fault in a network with a resonance- and short-term resistance-grounded neutral point -Electrical diagram; F ... fault location

According to Fig. 20, the earth-fault current  $I_F$  is composed of the capacitive current contribution of the individual sections of the network ( $I_{\text{grid, cap, ef}}$ ), the inductive coil current  $I_{\text{Pet}}$  and the current through the neutral-point resistance  $I_R$ .

When it comes to a real network technology, resistive current  $I_R$  becomes so high that the line impedance to fault point "F" can – in contrast to the previous section - no longer be neglected.

As in the previous sections, the method of symmetrical components is taken as a basis for a simplified presentation.

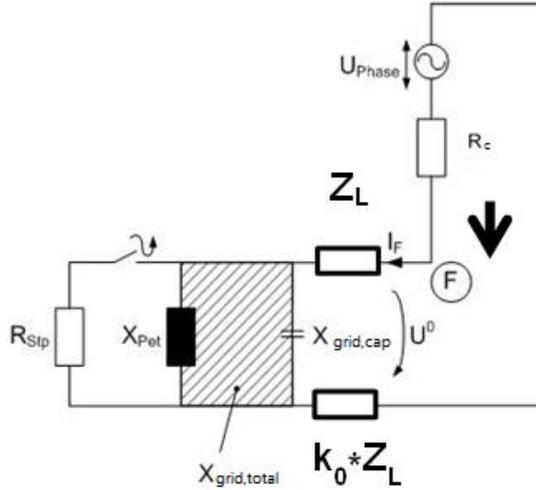


Fig. 20. Earth fault in a network with a resonance-g and short-term resistance-grounded neutral point–equivalent circuit diagram; F ... fault location

To answer the core question of the earth fault calculation, the following current loop has to be considered (see Fig. 20).

- substation busbar  $\rightarrow$  line-to-ground transition resistance  $\rightarrow$  phase conductor  $Z_L \rightarrow$  parallel connection of the source impedance of the network  $X_{grid,total}$  (network capacity // Petersen coil inductance) and the neutral point resistor  $R_{Stp} \rightarrow$  line-to-ground transition resistance  $R_c \rightarrow$  earth return path impedance  $Z_m = k_0 Z_L \rightarrow$  star point resistor  $R_{Stp} \rightarrow$  substation busbar

$$I_F = \frac{U_{Phase}}{(Z_L + R_c + Z_m + \frac{X_{grid,total} \cdot R_{Stp}}{X_{grid,total} + R_{Stp}})} \quad (10)$$

$$I_F = \frac{U_{Phase}}{\frac{R_{Stp} \cdot X_{grid,total}}{R_{Stp} + X_{grid,total}} + Z_L \cdot (1 + k_0) + R_c} \quad (11)$$

$R_{Stp}$  ... star-point resistor

$X_{grid,total}$  ... source impedance of the network

$$X_{grid,total} = \frac{U_{Phase}}{m \cdot I_{grid,cap,ef}} \quad (12)$$

$m$  ... degree of the network overcompensation in p.u.

$Z_L$  ... impedance of the phase conductor,

$Z_m$  ... earth return path impedance of the “ground”

$R_c$  ... line-to-ground transition resistance at the fault location

$k_0$  ... Ratio between the earth return-path impedance and the impedance of the phase conductor

## 10 CONCLUSION

The paper proposes a procedure to calculate the earth-fault current, for the medium-voltage network with a tolerable imprecision.

Using these attained simplified calculations makes the task of determination of the earth-fault currents considerably easier compared to the other approaches.

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