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Equivalent Circuit and Induction Motor Parameters for Harmonic Studies in Power Networks

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Abstract: *An* induction motor model for high harmonics used in literature for harmonic penetration studies and the corresponding parameters are analyzed in this paper. It is shown that the rotor slot resistance and the rotor slot reactance values are equally in the short circuit regime and that they have practically equal values, in per unit, for all motors of the same series. The knowledge of the equivalent circuit parameters of the squirrel-cage induction motors operating in the short circuit regime is especially important when calculating the current power losses in induction motors supplied from power set with high harmonics. General conclusion is that harmonic reactance values used in literature are more or less accurate (or by 10% higher). On the contrary, real resistance values are 2~4 times higher and the active powers absorbed by the rotation machine do not exactly correspond to the damping value. Consequently, the amplification factor of the source harmonic current calculated in literature for resonant regimes is 2~4 times higher than the real values.

Keywords: Induction motors, high hamonics, eequivalent circuit, parameters.

1 INTRODUCTION

The goal of our paper is to improve the induction motor model for high harmonics used in harmonic penetration studies and corresponding parameters calculation. The induction motor parameters and particularly the rotor resistance are determined and analyzed.

1.1 Equivalent circuit of motor for high harmonics

Saving the electric energy in the drive can be achieved by improving the power quality in the consumer power network. Electric energy consumption in the plant can be reduced by improving the quality of AC power consumers. The term power quality [1-5] mainly involves the quality of the supply voltage, which meets the prescribed criteria in respect of :

- voltage level tolerances within UN±5%,
- limit voltage harmonic distortion of THDU≤3÷8% (higher values refer to a network of a lower voltage level), and the limit voltage unbalance of 2%, its impact on the motor own and economical operation is considerable.

Nowadays, the presence of high harmonics in the supply voltage is often due to the increased number of consumers supplied by the rectifiers and invertors.

1.2 Equivalent circuit of motor for high harmonics

The voltage higher harmonics on the motor connection may exist for two reasons [1]:

- motor is supplied by a variable frequency converter,
- presence of higher harmonics in the network is result from other nonlinear loads.

Namely, while plants with the frequency and speed regulators are taking measures towards reducing power losses, motors designed to operate at a fixed frequency are exposed to increased power losses when there are higher harmonics in the voltage waveform. In the paper, the latter case is considered, i.e. the impact of higher voltage harmonics on the motor parameters and characteristics. The first to be dealt with is the equivalent circuit with appropriate parameters for the higher harmonics. Knowing the value of resistance ($R_{M,h}$), reactance ($X_{M,h}$) and motor impedance ($Z_{M,h}$) for the higher harmonics, the total resistance, reactance and impedance of the network for higher harmonics can be determined. This is particularly important for a network with capacitors for reactive power compensation when there is a possibility of resonance [4, 5]. In these regimes, the impact of the impedance on the overall impedance of the motor network is increased and may sometimes be the reason for the occurrence of the resonance phenomenon.

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The equivalent circuit of the induction motor for harmonics of a higher order of harmonic ($h = f_h/f_1$) is similar to the one of the induction motor in the short circuit regime with its rotor locked [6]. The motor slip, compared to the harmonic rotating fields, is given by s_h=1± (1-s)/h, where *h* is the motor slip in relation to the first harmonic field. So, when the motor operates in its normal regime, its slip is $s = 0.01 \div 0.06$, for higher harmonics it is $s_h = 1 \pm 1/h \approx 1$ and in the starting mode $s_h = 1$. The skin effect increases the rotor conductor resistance and reduction of the rotor's conductor inductance [7]. The effect is similar to that of the short circuit motor mode (with the rotor locked), but the increase in the resistance and the decrease in the inductance are even greater since the frequencies of the induced currents in the rotor chamber are several times larger, $f_2 = (h \pm 1)\ f1 >> 1$ for $h \geq 5$ [8].

To determine the resistance and reactance of motor equivalent circuit for higher harmonics, an appropriate circuit for the induction motor in the short circuit regime should be selected. But in literature and in practice this is often done differently. When modeling an induction motor in order to calculate the power network higher harmonics:

- resistance of the motor is modeled together with the total consumer active power in the network, though it is apparent [5] that it does not correspond with the impact of motor resistance on the suppression of higher harmonics, and
- equivalent reactance is estimated based on the number and (installed) motor power in operation and the motor's inverse reactance, which is multiplied by the number $h = h/f_1$ (*h*-order of the harmonic).

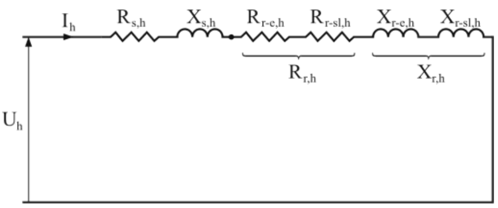
As the frequency of the induced currents in the rotor is *h*-times higher, compared to the frequency of the induced currents in the rotor of the motor in short circuit regime, from the first to be dealt with are the parameters and equivalent schemes for induction motor operating in the short circuit regime. Besides the obvious increase in accuracy, the procedure will not be more complicated, because in the paper [9-11] developed method is presented for calculating the parameters of the motor equivalent schemes under short circuit on the basis of the manufacturer's data. These will be shortly presented and used to determine the parameters of the higher harmonics equivalent scheme.

2 Equivalent Circuit and Motor Parameters for Harmonics

The first to be determined for a given equivalent circuit for the harmonics are the motor parameters.

2.1 Equivalent circuit of the motor for high harmonics

The equivalent circuit for the higher harmonics is identical to that for the short circuit mode for primary frequency f_1. The values of the rotor resistance components ($R_{r-e,h}$), rotor slot resistance ($R_{r-sl,h}$) and rotor end resistance ($R_{r-e,h}$) and rotor reactances ($X_{r-e,h}$, $X_{r-sl,h}$) for the higher harmonics are presented in the Fig. 1a [10, 11].

a)


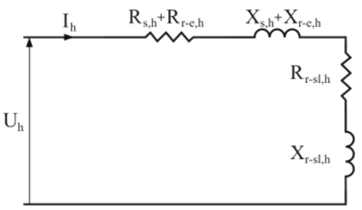
b)


Figure 1. Motor equivalent circuit for higher harmonics: a) with separate $R_{r-e,h}$, $R_{r-sl,h}$ and $X_{r-e,h}$, $X_{r-sl,h}$ and b) with grouped resistances and reactances.

2.1 Induction Motors Operating in a Network with Harmonics

The skin effect is practically expressed only on the part of the conductor in the rotor slot , i.e. it only to increases the rotor slot resistance ($R_{r-sl,h}$) and reduces rotor slot inductance ($L_{r-sl,h}$). As a result, the values of the rotor slot resistance and rotor slot reactance become equal at relative penetration depth $\zeta = H/\partial \geq 1.5$ [2]-[4]. As the penetration depth is, already for the fifth harmonic, $\partial_{Al,h=5} = 4.5$ mm, $R_{r-sl,h}$ always equals to $X_{r-sl,h}$ ($R_{r-sl,h} = X_{r-sl,h}$), Fig. 2. For this reason, similarly as for the corresponding scheme for the short-circuit mode, the rotor reactance ($X_{r,h}$) and the rotor resistance ($R_{r,h}$) are divided into two components in the equivalent circuit for the higher harmonics [4-5]:

$$R_{r,h} = R_{r-sl,h} + R_{r-e,h} \tag{1}$$

$$X_{r,h} = X_{r-sl,h} + X_{r-e,h} \tag{2}$$

As the resistance and reactance of the stator windings and resistance and reactance of the rotor conductors outside the slots are grouped (Fig.1b), the impact of the skin effect can be neglected. These values remain unchanged in both in the nominal regime and in the short circuit regime. They are collective resistance $R_{s,h} + R_{r-e,h}$ and reactance, $X_{s,h} + X_{r-e,h}$. Remaining resistance $R_{r\text{-}sl,h}$ and reactance $X_{r\text{-}sl,h}$, are separated on the equivalent circuit, Fig. 1b.

2.2 Equivalent circuit of the motor for high harmonics

The knowledge of the equivalent circuit parameters of the squirrel-cage induction motors operating in short circuit regime is especially important when calculating the current in a power set with the voltage high harmonics.

The current frequencies of individual harmonics in the rotor winding are h-times higher ($f_{r,h} \approx h{\cdot}f_1 = h{\cdot}f_{r,SC}$), so the theoretical depth of penetration (∂_h) of these currents is $\sqrt{h}$ -times lower. The explanation is based on the fact that the motors of the power above 3 kW (or with a relative penetration depth $\zeta_{ScC} = H/\partial_{SC} \geq 1.2$), already in the short circuit mode ($f_{SC} = f_1$), the whole section (or height H) of the rotor bar [9] is not used, the actual depth of penetration of individual harmonic currents (∂_h) is thus $\sqrt{h}$ -times lower. This means that the corresponding section of rotor conductor is $\sqrt{h}$ -times lower, so the values of the rotor slot resistance are $\sqrt{h}$ times higher and values of rotor slot inductance are $\sqrt{h}$ times lower compared to the values of the fundamental harmonic in the short circuit regime. In general, rotor slot resistance ($R_{r\text{-}sl,h}$), rotor slot inductance ($L_{r\text{-}sl,h}$) and rotor slot reactance ($X_{r\text{-}sl,h}$) in the function of harmonic order $h = f/f_1$ (i.e. the relative depth of penetration $\zeta_h = H/\partial_h$) are shown in Fig. 2.

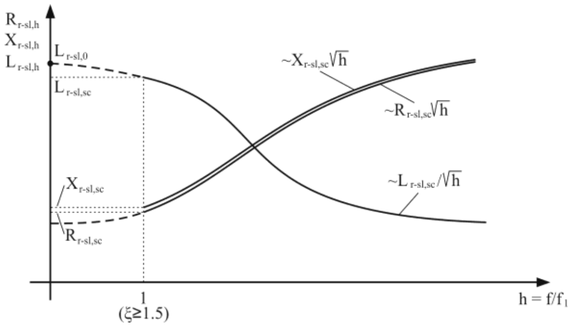


Figure 2. Rotor slot: resistance ($R_{r\text{-}sl,h}$), inductance ($L_{r\text{-}sl,h}$) and reactance ($X_{r\text{-}sl,h}$) dependencies of harmonic order $h = f/f_1$

As the depth of penetration $\partial_{Al,h=5} = 4.5$ mm and the relative penetration depth are equal to $\xi_h = H/\delta_h \geq 2$ for the harmonics of the order $h \geq 5$, the skin effect is practically expressed $X_{r\text{-}sl,sc} = R_{r\text{-}sl,sc}$. For the motors of the power above 3 kW ($H_{Al} \geq 15$ mm), for parameters of the motor for high harmonics, the following equations can be written:

- for the rotor slot resistance

$$R_{r-sl,h} = R_{r-sl,sc} \cdot \sqrt{h} \tag{3}$$

- for the rotor slot inductance

$$L_{r-sl,h} = L_{r-sl,sc} / \sqrt{h} \tag{4}$$

- for the rotor slot reactance

$$X_{r-sl,h} = X_{r-sl,sc} \cdot \sqrt{h} \tag{5}$$

The total values of motor resistance ($R_{M,h}$), reactance ($X_{M,h}$) and motor impedance ($Z_{M,h}$) for higher current harmonics, are:

$$R_{M,h} = \left(R_s + R_{r-e}\right) + R_{r-sl,sc} \cdot \sqrt{h} \tag{6}$$

$$X_{M,h} = \left(X_s + X_{r-e}\right) \cdot h + X_{r-sl,sc} \cdot \sqrt{h} \tag{7}$$

$$Z_{M,h} = \sqrt{R_{M,h}^2 + X_{M,h}^2} \tag{8}$$

As seen from [9] the rotor slot resistance ($R_{r\text{-}sl,sc}$) and rotor slot reactance ($X_{r\text{-}sl,sc}$) values in the short circuit regime are approximatively the same for the motors of any powers in a given series:

$$R_{r-sl,sc} = R_{r,sc} - R_{r-e,sc} \approx Const \tag{9}$$

$$X_{r-sl,sc} = X_{r,sc} - X_{r-e,sc} \approx Const. \tag{10}$$

Their values are within a narrow range of $X_{r\text{-}sl,SC}=R_{r\text{-}sl,sc} \approx 0.027 \div 0.033$ pu, and for the motors of large (>100 kW), medium (11÷50 kW) and low power (1÷7.5 kW), they are:

$$R_{r-sl,sc} = X_{r,sc} \approx 0.030\,pu \tag{11}$$

The total value of resistance ($R_s + R_{r\text{-}e}$) ≈Const is approximately the same value in any mode: either the operating regime, the short circuit regime and for regimes with harmonics. This applies to summary value of the related inductance of ($L_s + L_{r\text{-}e}$)≈*Const*. For the calculation values of $X_{M,h}$ and $R_{M,h}$ in equations (6) and (7), values (R_s+$R_{r\text{-}e}$) and (R_s+$R_{r\text{-}e}$) are determined with the following procedure.

According to the basic equations (6), (7) and (11), values (R_s+$R_{r\text{-}e}$) and (R_s+$R_{r\text{-}e}$) are determined with equations:

$$R_s + R_{r-e} \approx R_{M,SC} - 0.030\,(\text{pu}) \tag{12}$$

$$X_s + X_{r-e} \approx X_{M,SC} - 0.030\,(\text{pu}) \tag{13}$$

Based on the **locked rotor test,** or manufacturer's data, $i_{SC} = I_{SC}/I_N$ and $p_{SC} = P_{SC}/P_N$, the values of short circuit impedance ($Z_{M,SC}$) and short circuit resistance ($Z_{M,SC}$) are calculated:

$$Z_{M,SC} = 1/i_{SC} \tag{14}$$

$$R_{M,SC} = p_{SC}/i_{SC}^2 \tag{15}$$

The short circuit reactance value is calculated by equations:

$$X_{M,SC} = \sqrt{Z_{M,SC}^2 - R_{M,SC}^2} \quad (16)$$

or approximately

$$X_{M,SC} \approx 1/i_{SC} \quad (17)$$

$$X_{M,SC} = 4.5 \div 6\,pu \approx 5\,pu \quad (18)$$

2.3 Calculation and Analysis of the Harmonic Parameters for an Induction Motor Series

For the motor series ranging from 3 kW ÷ 400 kW, the values for stator resistance R_s =0.045Z_N÷0.015Z_N [13], and values of resistance and reactance for the short circuit regime, calculated in [4-5] are given. As evaluated in (11), the approximate value of $R_{ah} \approx 0.03 \cdot \sqrt{h}$.

The values of the motor parameters are calculated and given in Table 1:

- Resistance (R_{amah},), reactance (M_{Hz}) and impedance (M_{Hz}),
- Higher harmonic currents (M_{ish}), and
- ratio value M_{Hz}/R_{amah}.

3 RESONANT CIRCUIT MODEL FOR HARMONICS

Ratio values X_{Mh}/R_{Mh} are used to determine the harmonic currents in resonant regimes. In a resonant state, the bus capacitors combine with the system reactance to form a tank circuit, Fig. 3 [5], resulting in amplified currents of the resonant frequency flowing from the capacitor bank to the system reactance.

During resonance, too, the tank circuit, appearing as a high impedance anti-resonant (parallel) circuit (Fig. 4), is proposed for induction motors [4] and [14] though the active powers absorbed by the rotation machine do not exactly correspond to the damping value, i.e.

$$R_{M-p,h}(1) = \frac{V^2}{P_{50}} = \frac{V^2}{P_{SC}} = R_{M,SC} = Const \neq F(h) \quad (19)$$

as for the motors with the power r > 100 kW, $P_{SC} \approx P_R = P_{50}$.

Table 1: Values of: $R_{M,h}$, $X_{M,h}$, $Z_{M,h}$ and $X_{M,h}/R_{M,h}$, for motors > 100 kW (left) and 3-10 kW (right), for given harmonics (h) corresponding values $K_{2\square}$ and $K_{2\square,C}$

h=f/f1	R_s	$R_{r,h}$	$R_{M,h}$	$X_{M,h}$	$Z_{M,h}$	**$X_{m,h}/R_{m,h}$**	$X_{M,h}/R_{M,h}(1)$
1	0.015÷0.050	0.030	*0.045÷0.080*	0.161	0.167÷0.180	3.577÷2.006	3.577÷2.006
5	0.015÷0.050	0.067	0.072÷0.117	0.735	0.739÷0.744	10.020÷6.282	17.885÷10.030
7	0.015÷0.050	0.079	0.094÷0.129	1.018	1.022÷1.053	10.666÷7.898	25.039÷14.042
11	0.015÷0.050	0.099	0.114÷0.144	1.540	1.583÷1.586	13.544÷10.723	39.347÷22.066
13	0.015÷0.050	0.108	0.123÷0.158	1.811	2.419-÷2.421	14.756÷11.493	46.501÷26.078
17	0.015÷0.050	0.124	0.129÷0.174	2.694	2.643÷2.646	20.883÷15.460	60.809÷34.103
19	0.015÷0.050	0.131	0.146÷0181	3.249	3.252÷3.254	22.254÷17.353	66.477÷38.144
23	0.015÷0.050	0.144	0.159÷0.194	3.526	3.534÷3.536	22.176÷18.144	80.885÷46.138
25	0.015÷0.050	0.150	0.165÷0.200	3.833	3.836÷3.838	23.230÷19.150	88.039÷50.150
29	0.015÷0.050	0.162	0.177÷0.212	4.080	4.084÷4.086	23.005÷18.302	103.723÷58.174
31	0.015÷0.050	0.167	0.182÷0.217	4.357	4.361÷4.362	23.933÷20.083	110.833÷62.186
35	0.015÷0.050	0.177	0.192÷0.227	4.910	4.919÷4.920	25.099÷21.630	115.195÷70.210
37	0.015÷0.050	0.182	0.197÷0.232	5.187	5.191÷5.192	26.150÷22.344	132.349÷74.222

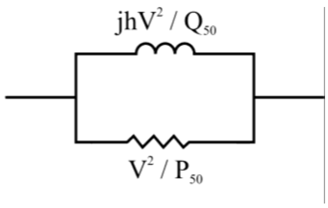


Figure 3. Induction motor model (parallel circuit) for harmonic penetration studies

The model obtained exactly from Fig.1 and shown in Fig. 4a, is transformed in to a parallel circuit, Fig. 4b (as in Fig. 3).

(a)

(b)

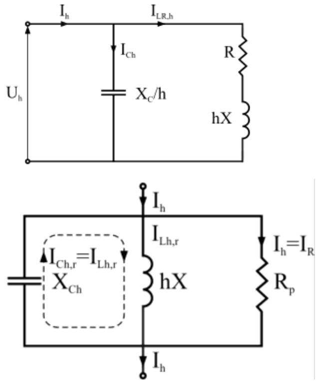


Figure 4. Resonant circuit model for harmonics: a) serious circuit, and b) corresponding parallel circuit

As for $X_{Mh} \geq 10R_{Mh}$ for the harmonic order of $h \geq 5$ (see Fig. 4a and Fig. 1), when the motor is modeled with a parallel circuit (see Fig. 4b), the reactance equals the value for the serious circuit, $X_{Mh} = R_{M\text{-}p,h}$.

$$R_{M-p,h} = (X_{Mh} / R_{Mh}) \cdot X_{Mh} \tag{20}$$

The amplification factor of the source current is given with the system X_L/R ratio times the harmonic order, i.e. it equals $h \cdot X_L/R$. The corresponding values of the $(X_{M,h}/R_{M,h})$ ratio, i.e. the amplification factor values of the source current, are given in Table 1, for the harmonics of the order h=5, 7, 11, 13, 35, 37. The values of the $(X_{M,h}/R_{M,h})$ ratio and reactance $X_{M,h}$ should be known, especially when the total reactance of all motors $\Sigma X_{M,SC} \leq (3{\sim}4)X_t$ (X_t – supplied transformer reactance). This condition is met when the total short circuit power of all the switched motors is $(\Sigma S_{M,,SC})$:

$$\Sigma S_{M,SC} \geq 0.3 S_{t,R} / U_{SC} \tag{21}$$

where:
S_{tR} – rated power of the supplied transformer,
U_{SC} - transformer short circuit voltage, in *pu*.

As the total rated power of all the switched motors $(\Sigma S_{M,R})$, with $\Sigma S_{SC}/\Sigma S_{M,SC,h}=5$ and U_{SC}=0.10pu=10, condition (21) may be expressed with the following equation

$$\Sigma S_{M,SC} \geq 0.3 S_{t,R} / U_{SC} \tag{22}$$

Condition (22) is often met when the MV motor drives supplied by transformers are $U_{SC} \geq 0.10$pu=10%.

For the $R_{M\text{-}p,h}(1)$ values for the model shown in Fig. 3 and determined by (19), the corresponding total series value of $R_{M\text{-}p,h}(1)$ is

$$R_{M-p,h}(1) = R_{M,SC} \prec R_{M-p,h} \tag{23}$$

The corresponding total series reactance values $X_{Mh}(1)$, for the model shown in Fig. 3, are:

$$X_{Mh}(1) = hX_{M,SC} \succ X_{Mh} = h(X_s + X_{r-e}) + \sqrt{h} X_{r-sl} \tag{24}$$

The corresponding values of the amplification factor are

$$\frac{X_{Mh}}{R_{M-p,h}(1)} = \frac{X_{Mh}}{R_{M-p,SC}} \succ \frac{X_{Mh}}{R_{M-p,h}} \tag{25}$$

The values $X_{Mh}/R_{Mh}(1)=h \cdot (X_{M,SC}/R_{M,Sc})$ are calculated and shown in Table 1. They are greater than the values of X_{Mh}/R_{Mh}
about 2~3 times for the harmonics $5 \leq h \leq 13$, and
about 3~4 times for the harmonics $17 \leq h \leq 25$.

4 Conclusion

An induction motor model for high harmonics used in harmonic penetration studies and its corresponding parameters are analyzed. General conclusion is that the harmonic reactance values are accurate (or by 10% higher). Whereas the resistance values are increased for 2 - 4 times. The amplification factor of the source harmonic current in resonant regimes is increased for the same number of times, as the real values.

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